Enforcement of Property Rights in a Barter Economy

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Abstract

If property rights to consumption goods are insecure, the incentives to trade in a barter economy are reshaped. In a pure exchange economy, we examine the case where two contestable consumption goods are vital to two agents and initial endowments follow a binary distribution. In line with the existing literature, we examine in a two–stage game how the equilibrium security of claims to property is determined. We find that, in equilibrium, two different regimes emerge, depending on the exogenous preference and appropriation-effectiveness parameters: *Peaceful coexistence* and *trade and appropriation*, with the former regime strictly Pareto-dominating the latter regime.

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1 Introduction

Traditionally, economic theory emphasized the mutual benefits arising from market transactions. One major assumption necessary for this theory was that individuals are rational and self interested. However, conflict, broadly defined as the use of resources for creating or defending against appropriation, is entirely absent in the Walrasian world. Together with the assumption of fully rational optimizers, this restriction boils down to the assumption that property rights are perfectly defined and costlessly enforced by an outside enforcement agency (*outside enforcement approach*)[1]. If property rights are imperfect, the potential for conflict arises, and with it an inefficient use of resources, such as the building of fences to guard initial claims or the use of time and effort to steal. Early models of conflict have concentrated on the effects of appropriative activity in a pure production/appropriation environment (Bush and Mayer (1974), Hirshleifer (1991) and Skaperdas (1992)). However, they have neglected the potential for mutually beneficial economic activity, such as the trade of goods, despite potential conflict. Embedding appropriation into economic modeling as an economic activity does not eliminate cooperation. In fact, if cooperation occurs, it does so in the shadow of conflict, which exposes the full costs of cooperation (in terms of endogenous transaction costs) and appropriation (in terms of forgone exchange possibilities). Recent articles make allowance for this insight.[2] In allowing the agents to decide whether they exchange goods and/or appropriate the claims of others, these authors incorporate what Hirshleifer (1994) called *the dark side of the force* into classic general equilibrium models.
One difference between these models lies in the fact that they consider *different categories of economic activity*: The models of Anderton and Carter (2004) and Skaperdas and Syropoulos (2002) include appropriation and exchange, whereas Anderson and Marcouiller (2005), Hausken (2004) and Rider (1999) include appropriation, production and exchange. Differences also arise with regard to *the factors which are subject to appropriation*: Rider (1999) assumes that, in a two-consumption-good, two-agent world, while inputs are unappropriable, *productions* are subject to appropriation. Each agent can invest effort in order to defend the homemade good or to appropriate the foreign good, prior to the exchange of goods, provided that each agent has the technology to produce one, and only one, of the two goods. Thus, the potential appropriation appears as an ex-ante reallocation of outputs. Using a ratio form of the contest-success function (CSF) he shows that uncontested exchange is impossible, i.e. exchange is always accompanied by appropriation efforts of both agents.[3] Hausken (2004) shows in a similar environment[4] that appropriation efforts become zero in equilibrium, if productions are sufficiently vulnerable to appropriation, i.e., appropriated goods may be partly destroyed.[5] Only in this case, due to

[1]See Hafer (2000).
[2]See Anderson and Marcouiller (2005), Anderton (2000), Anderton and Carter (2004), Hausken (2004), Rider (1999) and Skaperdas and Syropoulos (2002).
[3]Contested exchange are at hand, if the benefit parties derive from transactions are contingent on their abilities to enforce contracts. See Bowles and Gintis (1993).
[4]As in Rider (1999), there is a two consumption–good, two agent world. Each agent can produce only one good, only productions are subject to appropriation and potential appropriation is established prior to the trade of goods.
[5]Grossman and Kim (1995) make a similar assumption.

the zero value of appropriation, property rights are perfectly secure. Anderton and Carter (2004) present a model of vulnerable trade, in which the initial claims (two consumption goods) of two agents are secure by definition, but goods are subject to appropriation once they are offered for trade. They find that trade will only occur, if the exogenously given relative effectiveness of appropriation compared to defensive activities is low enough. Otherwise, both agents will decide to back off from trade (autarky). Thus, perfectly secure property rights only emerge if the factor that is subject to appropriation (exports) has a zero value.

We would like to present a model which includes appropriation and exchange in a two-good, two-agent environment where each agent has initial non-overlapping claims to property in the full stock of one consumption good. As in Rider (1999) and Hausken (2004), each agent can divert a fraction of a scarce resource in order to defend his initial claim or to appropriate the initial claims of the other, using a ratio form of the CSF. In the spirit of Grossman (2001), the security of claims to property depends on the fraction that each agent can successfully defend. This fraction depends on the relative allocation of an uncontestable and scarce resource (e.g. time) to both defensive activities (building a fortification) and appropriation activities (use of offensive weapons). The relative effectiveness of both activities is determined by an exogenous parameter.

We are especially interested in answering the question whether trade reduces the incentives to engage in the contest. Therefore, in contrast to the aforementioned literature, we seek to find out whether the *post-trade allocation of consumption goods* is perfectly secure. We assume that preferences towards both consumption goods are strictly convex and monotonous. Hence, trade not only reduces the amount of the good subject to appropriation but also decreases the marginal willingness to pay for it. The question we pose is: *Does the anticipation of potential appropriation force the agents to engage in trade (which mutually reduces the gains from appropriation) such that the resulting post-trade allocation of consumption goods is uncontested?*

As in Grossman and Kim (1995), we would additionally like to capture the role of defense as a deterrent to appropriation. Hence, we construct the following two-stage game: In the first stage, each agent decides simultaneously and independently on the amount of resource he wishes to sacrifice in order to defend his initial claim. Given these irreversible decisions, the agents, in the second stage, decide about the allocation of resource to appropriate the consumption good of the other agent as well as the amount of goods they wish to export. In contrast to Anderton and Carter (2004), goods designated for trade are no longer subject to appropriation. Thus, the consumables consist of the imported and appropriated fractions of the foreign good, and the successfully defended domestic non–exported goods. We would also like to examine the case where both consumption goods are vital to the agents. A quasi–linear utility function is assumed where both agents derive positive and decreasing marginal utility from both consumption goods. An exogenous parameter reflects the relative preference for both agents towards the consumption good in which they have initial claims to property.[6] Given these assumptions and the structure of the game, we find that, in equilibrium, two different regimes emerge, depending on the exogenous preference and appropriation effectiveness parameters:

[6] This means that both agents equally prefer the domestic good and equally prefer the foreign good, which defines a proper subset of the preferences suggested by Hausken (2004).

1. *Peaceful coexistence*: If the effectiveness of time allocated for appropriation - relative to the time allocated for defensive fortification - is sufficiently low, deterrence will result. Both agents invest no resource in appropriating the initial claims of others, and property rights are perfectly secure, a situation that never occurs in Rider (1999). Moreover, both agents trade goods efficiently. Since both consumption goods are vital to the agents, inefficiency arises caused by the investment necessary to deter the appropriation of non-exported goods. This finding emerges, in contrast to Hausken (2004), although appropriation is not destructive.

2. *Trade and Appropriation*: For higher relative effectiveness of appropriation, we observe the emergence of appropriation. Since the relative effectiveness of appropriation is heightened, the marginal benefit of challenging initial claims increases, thus encouraging agents to invest in appropriation. Trade of goods still exists, but the incentives to trade are violated: Both agents export a smaller amount than the amount they would export in a standard Walrasian economy.

The paper will proceed in the following manner: first, we will present the second–stage and the first–stage optimization problem of the two agents. Second, we will present the emerging equilibriums of the full game and the corresponding welfare aspects. Third, we will conclude.

2 Assumptions

Suppose that two agents derive utility from two divisible and contestable consumption goods and one non–appropriable resource, e.g. time. c_i^j defines agent i's consumption of the jth consumption good, with $i, j = 1, 2$. Each agent is endowed with Z units of one consumption good, i.e. agent i has initial nonoverlapping claims to Z units of the ith consumption good (Z^i). Moreover, each agent is endowed with L units of the resource. The vector of initial endowments is therefore $\boldsymbol{\omega} = (\boldsymbol{\omega}_1, \boldsymbol{\omega}_2)$, with

$$\boldsymbol{\omega}_1 = \begin{pmatrix} Z^1 \\ 0 \\ L \end{pmatrix}, \qquad \boldsymbol{\omega}_2 = \begin{pmatrix} 0 \\ Z^2 \\ L \end{pmatrix}. \tag{1}$$

Due to the initial endowments, if trade occurs, agent 1 will export good 1 while agent 2 will export good 2.[7] We will denote the amount of non–exported goods of agent i by z_i. Thus, $Z^i - z_i$ represents the exports of agent i. We assume that traded goods are unappropriable, which means that once agent i exports the amount $Z^i - z_i$, it is technologically impossible for him to regain these exports. Consequently, if trade occurs, all non–exported goods are subject to appropriation. If trade does not occur,

[7]For convenience we will talk about *exports* (*imports*) if the excess demand of agent i with regard to consumption good i is negative (if the excess demand of agent i with regard to the consumption good j is positive), with $i \neq j$. Moreover, we will talk about the *domestic* (*foreign*) good of agent i with regard to the ith consumption good (the jth consumption good), although this model does not necessarily apply to trade between states.

the whole initial endowment is challengeable.
The resource (L) can be allocated for defensive fortification (d_i) in order to defend all non-exported units of the initial claim, or it can be used for offensive weapons (a_i) in order to appropriate. Finally it can be used for leisure (l_i).

$$L \equiv a_i + d_i + l_i. \tag{2}$$

We assume that $L > a_i + d_i$, which means that we allow L to be sufficiently large so that corner solutions can be ruled out.
The fraction of the non–exported endowment of good i that agent i can successfully defend is represented by the Grossman modification of the Tullock CSF:

$$F_i^i(d_i, a_j, \theta) = \begin{cases} 1, & \text{for } a_j = 0, \\ \frac{d_i}{d_i + \theta a_j}, & \text{else,} \end{cases} \tag{3}$$

with $i \neq j$.[8] $F_i^i(d_i, a_j, \theta)$ is a function of the efforts of agents, raised in order to defend (d_i) and to appropriate (a_j) the initial claims of agent i. The exogenous parameter θ, with $\theta \in [0, 1]$, measures the effectiveness of time allocated for appropriation of initial claims, relative to time invested into defending initial claims.[9] For θ equal to zero, challenging initial claims is impossible. In this case $F_i^i = 1$, irrespective of the investment in appropriation, which represents one assumption in a standard Walrasian economy: perfectly secure and costlessly enforced property rights. For $\theta \in]0, 1[$, appropriation is no longer precluded but still inferior to defense, in terms of effectiveness. $\theta = 1$ represents a challenge technology that does not discriminate between protecting and seizing: No advantage emanates from the initial claims to a consumption good.[10] In order for F_i^i to be well defined, we assume that the defended fraction is 1 if agent j allocates no resources for appropriation.
To get a simple function of the challenged fraction of a consumption good, we define:

$$F_i^j(d_j, a_i, \theta) \quad = \quad 1 - F_j^j(d_j, a_i, \theta) = \begin{cases} 0, & \text{for } a_i = 0, \\ \frac{\theta a_i}{d_j + \theta\, a_i}, & \text{else.} \end{cases} \tag{4}$$

That is, F_i^j represents the fraction that agent i can successfully appropriate of the consumption good j.[11]
In line with the literature on emerging property rights (see e.g. Grossman (2001)), the formal design allows us to distinguish between two different levels of property rights:

[8]See Tullock (1980) and Grossman and Kim (1995). Subscripts represent agents, while superscripts represent goods.
[9]It may represent formal property rights or it may reflect some technological gap between appropriating and defending initial claims. See Grossman (2001).
[10]The restriction of θ for being, at most, equal to 1 is to exclude the possibility of giving away one's own endowment in order to retain, ceteris paribus, the same amount at lower costs.
[11]Since only agent j has initial claims to consumption good j and exports are non–appropriable, this is equivalent to the statement that $F_i^j(d_j, a_i)$ is the fraction that agent i can successfully appropriate *from agent j*.

Definition 1
(*Level of security of property rights*)

1. *Property rights of agent i are said to be* **insecure**, *for $F_i^i < 1$ in equilibrium.*
2. *Property rights of agent i are said to be* **perfectly secure**, *if $F_i^i = 1$ in equilibrium.*

This means that the degree of security of claims to property is defined by the fraction of the initial–endowment good that each agent can successfully defend.
As has been mentioned above, both agents derive utility from the two consumption goods and leisure $(u_i(c_i^i, c_i^j, l_i))$. We assume that preferences are homothetic with regard to both consumption goods, and that each agent derives positive and non–increasing utility from both consumption goods and leisure.
Given the assumption that traded goods are not subject to appropriation, we define:

$$c_i^i(d_i, a_j, z_i) = F_i^i(d_i, a_j)\, z_i, \tag{5}$$

$$c_i^j(d_j, a_i, z_j, \tilde{z}_j) = F_i^j(d_j, a_i) z_j + \tilde{z}_i. \tag{6}$$

Equation (5) represents the consumption of the domestic good of agent i. It consists of the successfully defended fraction of the domestic good that agent i does not export. Equation (6) represents the amount of the jth good that agent i can consume. It consists of two parts, first part being the fraction of the unexported amount of the initial–endowment–good of agent j, that agent i appropriates $(F_i^j(d_j, a_i)z_j)$, and second part being the unappropriable imports of agent i $(\tilde{z}_i)$.
If we include the budget constraint of each agent, we can see that

$$\tilde{z}_i \quad = \quad \frac{(Z^i - z_i)p^i}{p^j}, \tag{7}$$

that is, the imports of agent i have to equal the exports of agent i evaluated by $\frac{p^i}{p^j}$ in equilibrium.
Again, both agents act in a two–stage game: In the first stage, agents decide independently and simultaneously on the level of defense (d_i). In the second stage, both agents decide independently and simultaneously on the level of appropriation (a_i) and on the amount of exported goods $(Z^i - z_i)$.

3 Appropriation and Trade

Given the level of defense of the first stage (d_1, d_2), each agent maximizes his utility in the second stage under certain constraints:

$$\begin{aligned} &\max_{a_i, z_i} \quad u_i\Big(c_i^i, c_i^j, l_i\Big) \\ &\text{s.t.} \quad a_i \geq 0, \quad z_i \geq 0, \quad Z^i - z_i \geq 0. \end{aligned} \tag{8}$$

The Lagrangian is

$$\mathcal{L}_i = u_i\Big(c_i^i, c_i^j, l_i\Big) + \kappa_i\, a_i + \lambda_i\, z_i + \mu_i(Z^i - z_i), \tag{9}$$

with κ_i, λ_i and μ_i representing the shadow prices for violating the non–negativity constraints.
The first-order conditions (FOCs) are:

$$\frac{\partial u_i(\cdot)}{\partial c_i^j}\frac{\partial F_i^j(\cdot)}{\partial a_i}\,z_j - \frac{\partial u_i(\cdot)}{\partial l_i} + \kappa_i \;= 0, \tag{10}$$

$$\kappa_i = 0 \quad \Longrightarrow \frac{\frac{\partial u_i}{\partial l_i}}{\frac{\partial u_i}{\partial c_j^i}} \;= \frac{\partial F_i^j(\cdot)}{\partial a_i} z_j, \tag{10'}$$

$$\frac{\partial u_i(\cdot)}{\partial c_i^i}\,F_i^i(\cdot) - \frac{\partial u_i(\cdot)}{\partial c_i^j}\,\frac{p^i}{p^j} + \lambda_i - \mu_i \;= 0, \tag{11}$$

$$\lambda_i = \mu_i = 0 \quad \Longrightarrow \frac{\frac{\partial u_i}{\partial c_i^i}}{\frac{\partial u_i}{\partial c_i^j}} \;= \frac{p^i}{p^j\,F_i^i(\cdot)}. \tag{11'}$$

These partial derivatives capture the key tradeoff for both agents. The first term on the left hand side (LHS) of equation (10) represents the gain in utility triggered by a marginal increase in appropriation, where $\partial F_i^j(\cdot)/\partial a_i \times z_j$ represents the marginal revenue of appropriation to agent i. The second term represents the loss in utility through a marginal increase in appropriation caused by the leisure time foregone. The last term on the LHS of (10) shows the positive shadow price for violating the non–negativity constraint on a_i.
Equation (10′) shows the FOCs for $\kappa_i = 0$, i.e. if the restriction $a_i \geq 0$ is *not binding*. Then, in optimum, the marginal rate of substitution (MRS) between the consumption of the foreign good and leisure (LHS of (10′)) equals the marginal revenue from appropriation (right hand side (RHS) of (10′)).
The first term on the LHS of (11) represents the gain due to a marginal increase of z_i. Since only the successfully defended fraction of z_i is consumable for agent i, the marginal utility in the domestic consumption good is weighed by the defended fraction ($F_i^i(\cdot)$). The second term represents the marginal loss in utility due to the reduced exports ($Z^i - z_i$), weighed by the terms–of–trade ($\frac{p^i}{p^j}$). The third and fourth term on the LHS of (11) show the shadow–prices for violating the non–negativity constraints $z_i \geq 0$ and $Z^i - z_i \geq 0$.
Equation (11′) represents the FOCs for $\lambda_i = \mu_i = 0$, i.e. if the restriction $z_i \in [0, Z^i]$ is *not binding*. The LHS of (11′) represents the MRS between both consumption goods, the RHS shows the relative price in optimum, which is the quotient of the price of the foreign good (p^j) and the *effective* price of the domestic good ($\frac{p^i}{F_i^i(\cdot)}$).[12]
As has already been mentioned, we are especially interested in analyzing the effects of the assumptions on an economy where both consumption goods are vital to the agents. Therefore, agents are forced to either participate in barter exchange, to appropriate, or both. The supposed utility function is assumed to satisfy the *Inada*-conditions:

$$\left.\frac{\partial\, u_i(\cdot)}{\partial c_i^j}\right|_{c_i^j \to 0} \to \infty. \tag{12}$$

[12]For $F_i^i = 1$, the marginal rate of substitution between both consumption goods equals the equilibrium terms–of–trade. For $F_i^i < 1$, it exceeds the equilibrium terms–of–trade, which captures the increased scarcity of the domestic good to agent i because the insecure property rights.

For tractability, the utility function is assumed to take the following form:

$$u_i(c_i^i, c_i^j, l_i) = \alpha \ln \left[c_i^i\right] + (1-\alpha) \ln \left[c_i^j\right] + l_i, \tag{13}$$

with $\alpha \in]0, \hat{\alpha}[$, $\hat{\alpha} = \frac{2}{2}$ and $l_i = L - a_i - d_i$.[13]

We assume that the amount of the initial–endowment good is identical, that is $Z^1 = Z^2 = Z$. Furthermore, we set $p^1 = 1$ and $p^2 = p$. At first we will take a look at the Nash-equilibria in the second stage, if the restrictions on a_i ($a_i \geq 0$) and z_i ($z_i \in [0, Z]$) are not binding, which we will call from now on the interior solution.

3.1 Interior solution

Given that $\kappa_i = \lambda_i = \mu_i = 0$ in equilibrium, we get the following values:

$$a_i(d_i, d_j) = \frac{(-2+\alpha)\, d_j - \alpha \sqrt{d_i\, d_j} + \sqrt{\alpha\, d_j}\sqrt{\alpha \left(\sqrt{d_i} - \sqrt{d_j}\right)^2 + 4\theta}}{2\,\theta}, \tag{14a}$$

$$F_i^i(d_i, d_j) = \frac{2d_i}{\alpha \left(d_i - \sqrt{d_i\, d_j}\right) + \sqrt{\alpha\, d_i \big(\alpha(d_i - d_j)^2 + 4\,\theta\big)}}, \tag{14b}$$

$$z_i(d_i, d_j) = \frac{\big(-1 + \alpha\, F_j^j(d_i, d_j)\big)\, \alpha\, Z}{-1 + \alpha - \alpha^2\, F_i^j(d_i, d_j)\, F_i^i(d_i, d_j)}, \tag{14c}$$

$$p\,(d_1, d_2) = \frac{-1 + \alpha - \alpha^2\, F_2^2(d_2, d_1)\, F_2^1(d_1, d_2)}{-1 + \alpha - \alpha^2\, F_1^1(d_1, d_2)\, F_1^2(d_2, d_1)}, \tag{14d}$$

with $i \neq j$ for $d_i, d_j > 0$.[14]

To illustrate the exchange of goods for an interior solution in the second stage, we take a look at figure 1, which represents the consumption space by the use of an Edgeworth-Bowley box. We assume $\alpha < \frac{1}{2}$ and $d_1 = d_2 > 0$. Given that both agents invest no time in appropriation ($a_1 = a_2 = 0$), property rights are perfectly secure. Hence, the offer–curve, which represents the set of foreign goods being demanded as the relative price varies, takes the typical form for Cobb–Douglas preferences ($\widetilde{OC}_i$ for agent i).[15] Thus, the intersection of $\widetilde{OC}_1$ and $\widetilde{OC}_2$ (point C), which graphically represents the equilibrium with regard to the traded consumption goods, lies on the contract curve (CC).

Given a strictly positive and identical investment in appropriation ($a_1 = a_2 > 0$), the offer–curve changes (OC_i for agent i). Since in this situation $F_1^1 = F_2^2 < 1$, the effective price of the domestic good rises (see ($11'$)). Hence, the incentives to trade are reshaped. Taking into account that property rights are insecure, each agent's amount of exported goods declines for every possible value of the terms-of-trade, compared to the first scenario.[16] Thus, the equilibrium value of exports

[13] The reason for the upper boundary of α is that for $\alpha > \hat{\alpha}$, the possibility of market breakdown emerges in equilibrium. This yields enormous analytical problems, however does not alter the main findings of the model.

[14] All calculations can be found in the appendix which will be sent to the reader upon request.

[15] This form of the offer–curve only emerges, if – as assumed – each agent only posses *one* of the consumption goods.

[16] Note that OC_1 never intersects $\widetilde{OC}_1$, i.e., it always lies to the right of OC_1. The same applies to OC_2: It never intersects $\widetilde{OC}_2$.

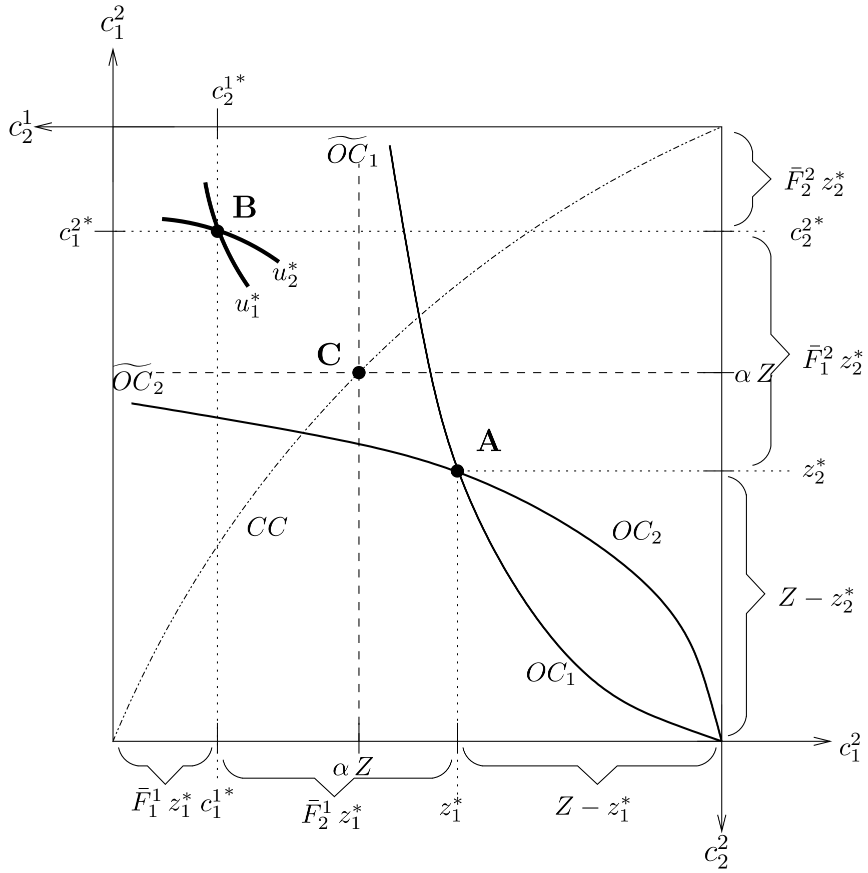

Figure 1: Determining the equilibrium level of exports ($Z - z_i^*$) in the case of an interior solution in the second stage.

($Z - z_1^*$ and $Z - z_2^*$, Point A in figure 1) declines. Given these exports, agent 1 appropriates the fraction F_1^2 of the non–exported goods of agent 2 (z_2^*), and agent 2 appropriates the fraction F_2^1 of the non–exported good of agent 1 (z_1^*). Thus, the equilibrium value of the consumption of the foreign good is the sum of imported and appropriated goods for both agents (c_1^{2*} and c_2^{1*}). The consumption of the domestic good in equilibrium equals the successfully defended fraction of the unexported goods (c_1^{1*} and c_2^{2*}). This determines the equilibrium level of consumption, which is graphically represented by Point B. Since property rights are insecure, the MRS of agent 1 and 2 with regard to the consumption goods are unequal in equilibrium: $MRS_1(c_1^{2^*}, c_1^{1^*}) > MRS_2(c_2^{2^*}, c_2^{1^*})$, thus representing the fact that not all gains from trade are exhausted due to the insecure property rights.

3.2 First corner solution

In the first corner solution, both individuals invest no time in appropriation ($a_i = 0$) and the restriction $z_i \in [0, Z]$ is not binding.

$$a_i = 0, \tag{15a}$$

$$F_i^i = 1, \tag{15b}$$

$$z_i = \alpha\, Z, \tag{15c}$$

$$p = 1, \tag{15d}$$

for $d_i \geq \alpha\,\theta$. That is, a corner solution concerning the level of appropriation is only consistent with an export level of $(1-\alpha)Z$ and a level of defense in the first stage of at least $\alpha\,\theta$. In this case, property rights are perfectly secure and efficiency arises with regard to the consumption goods (cf. equation (11′))

Definition 2
(*C–efficiency*)
An allocation is c-efficient if it is efficient in terms of the consumption goods:

$$MRS_1(c_1^{2^*}, c_1^{1^*}) = MRS_2(c_2^{2^*}, c_2^{1^*}).$$

Thus, in this case, the amount of exported goods is equal to the amount exported in a standard Walrasian economy. The fact that the equilibrium allocation is not *purely efficient* is worth noting. An efficient allocation of goods and resources would imply *c-efficiency* and the allocation of *no* resources among appropriative and defensive activities.[17]

3.3 Second corner solution

In the second corner solution, the trade volume drops to zero ($z_i = Z$) and the restriction $a_i \geq 0$ is not binding. Thus, the following reaction functions apply:

$$a_i = \frac{-d_j + \sqrt{d_j\big(d_j + 4(1-\alpha)\,\theta\big)}}{2\,\theta}, \tag{16a}$$

$$F_i^i = \frac{2\,d_i}{d_i + \sqrt{d_i\big(d_i + 4(1-\alpha)\,\theta\big)}}, \tag{16b}$$

$$z_i = Z, \tag{16c}$$

for $d_i \leq \frac{\theta(1-2\,\alpha)^2}{\alpha}$ and $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$. As we can see, this corner solution only emerges for a proper subset of α. The intuition behind this finding is the following: If the relative preference towards the domestic good (α) is strong enough, opportunity costs of imports of the foreign good are relative high. If, in addition, the level of defense in the first stage is sufficiently low ($d_i \leq \frac{\theta(1-2\,\alpha)^2}{\alpha}$), the opportunity costs of appropriation are lower than the opportunity costs of trade. In this case, barter exchange will vanish.
The following proposition recapitulates our findings.

Proposition 1
(*Barter Exchange and appropriation*)
Assuming symmetry with regard to the investment in defense ($d_1 = d_2 = d$), we find that

1. *If $d_1 = d_2 \leq \underline{d} = \frac{\theta\,(1-2\alpha)^2}{\alpha}$ and $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$, barter exchange will collapse ($z_i = Z$) in equilibrium. Moreover, property rights will become insecure ($F_i^i < 1$) due to the strict positive investment in appropriation ($a_i > 0$).*

[17] Since $d \geq \alpha\,\theta$ and $\alpha \in]0, \hat{\alpha}[$, this is only possible for $\theta = 0$, that is, if appropriation is technically impossible.

2. *If $d_1 = d_2 \geq \bar{d} = \alpha\,\theta$, appropriation will be deterred ($a_i = 0$). Thus, property rights are perfectly secure ($F_i^i = 1$) and barter exchange exhibits c–efficiency, i.e. $MRS_1(c_1^{2^*}, c_1^{1^*}) = MRS_2(c_2^{2^*}, c_2^{1^*})$.*

3. *If $d_1 = d_2 \in \left[\underline{d}, \bar{d}\right]$, neither of the restrictions ($z_i \in [0, Z]$ and $a_i \geq 0$) is binding.*

4 Defending

In the first stage, both agents decide simultaneously and independently on the time they wish to sacrifice for defensive actions (d_i). At first we would like to examine the case when the restrictions on a_i and z_i are not binding (interior solution). Given the values $a_i(d_i, d_j)$, $z_i(d_i, d_j)$, $F_i^i(d_i, d_j)$, and $p(d_1, d_2)$ (equations (14)), the indirect utility functions yield:

$$\begin{aligned} v_1(d_1, d_2) \ =& \alpha \ln\left[F_1^1(d_1, d_2)\, z_1(d_1, d_2)\right] \\ &+ (1-\alpha) \ln\left[F_1^2(d_1, d_2)\, z_2(d_1, d_2) + \frac{(Z - z_1(d_1, d_2))}{p(d_1, d_2)}\right] \\ &+ L - a_1(d_1, d_2) - d_1, \end{aligned} \tag{17.1}$$

and

$$\begin{aligned} v_2(d_2, d_1) \ =& \alpha \ln\left[F_2^2(d_1, d_2)\, z_2(d_1, d_2)\right] \\ &+ (1-\alpha) \ln\left[F_2^1(d_1, d_2)\, z_1(d_1, d_2) + \big(Z - z_2(d_1, d_2)\big)\, p(d_1, d_2)\right] \\ &+ L - a_2(d_1, d_2) - d_2. \end{aligned} \tag{17.2}$$

The aim of each agent is to maximize utility over d_i:

$$\begin{aligned} \max_{d_i} \quad & v_i(d_i, d_j) \\ \text{s.t.} \quad & d_i \geq 0. \end{aligned}$$

In analyzing the optimal level of defense in stage one, we can make use of the envelope theorem, which allows us to only consider the effect of a change in d_i on the functions that are parameters for agent i.[18] We are able to show that both FOCs are symmetric. Therefore, we know that $d_1 = d_2 = d$ is a Nash-equilibrium. Hence, the following FOC has to be satisfied:

$$H(\alpha, \theta, d) \stackrel{!}{=} 0, \tag{18}$$

[18]This is unprohibited since we assume that the restrictions on z_i and a_i are *not* binding.

with

$$
\begin{aligned}
H(\alpha,\theta,d) =& \left.\frac{\partial v_i}{\partial d_i}\right|_{d_i=d_j=d} \\
=& -1+\frac{\alpha\left(2-\sqrt{\frac{\alpha\, d}{\theta}}\right)}{4\, d} \\
&+\frac{\alpha^2\left(1-\alpha+\sqrt{\frac{\alpha\, d}{\theta}}\right)\left(d\left(2-\alpha+\sqrt{\frac{\alpha\, d}{\theta}}\right)-2\,\alpha\sqrt{\alpha\, d\,\theta}\right)}{4\left((1-\alpha)\sqrt{\alpha\,\theta\, d}+\alpha\, d\right)^2}
\end{aligned}
\tag{19}
$$

for $d > 0$.[19] We are able to show, that as long as $\theta > \widetilde{\theta} = \frac{\alpha}{4}$, there is a unique solution to this FOC in terms of d, which we will call d^I.
Given the above findings, we are able to draft the following proposition:

Proposition 2
(*Defending*)

1. *If $\theta \in [0,\bar{\theta}]$, both agents will choose the level of defense* **just sufficient** *to deter appropriation ($d = \bar{d}$), with $\bar{\theta} = \frac{1}{4}\left(2+\alpha-2\,\alpha\right)^2$.*
2. *If $\theta \in]\bar{\theta},1]$, both agents will choose a level of defense (d) which is* **insufficient** *to deter appropriation but which does not foreclose barter exchange ($d = d^I \in]\underline{d},\bar{d}[$).*
3. *Given the assumed set of exogenous preferences ($\alpha \in]0,\hat{\alpha}[$ and $\theta \in [0,1]$) the collapse of barter exchange never occurs.*

The proof of proposition 2.3 is left to the appendix.

Proof of proposition 2.1. and 2.2.
Since $d_1 = d_2 = d > \underline{d}$ in equilibrium the reaction functions of the second stage yield:

$$
a(d) = \begin{cases} \frac{\sqrt{\alpha\, d\,\theta}-d}{\theta} & \text{for } d \in]0,\bar{d}\,[, \\ 0 & \text{for } d \geq \bar{d}, \end{cases} \tag{20a}
$$

$$
F_i^i(d) = \begin{cases} \sqrt{\frac{d}{\alpha\,\theta}} & \text{for } d \in]0,\bar{d}[, \\ 1 & \text{for } d \geq \bar{d}, \end{cases} \tag{20b}
$$

$$
z(d) = \begin{cases} \frac{\alpha\sqrt{\theta} Z}{\sqrt{\alpha\, d}+(1-\alpha)\sqrt{\theta}} & \text{for } d \in]0,\bar{d}\,[, \\ \alpha\, Z & \text{for } d \geq \bar{d}, \end{cases} \tag{20c}
$$

$$
p = 1, \tag{20d}
$$

where the upper case is simply a reformulation of the reaction functions in the interior solution for $d_1 = d_2 = d$ (cf. equations (14)). The lower case corresponds

[19] Assuming that both individuals do not invest in defense ($d = 0$) is never a solution, irrespective of the level of θ and α. This is due to the fact that the benefit of a marginal increase in d goes to infinity for $d = 0$.

to the reaction functions in the first corner solution (cf. equations (15)), and since $d_1 = d_2 = d > 0$, the terms-of-trade in equilibrium equals one regardless of the level of d.

Equation (20c) tells us that the exported amount of the initial endowment good is strictly positive for all $d > 0$. Moreover, for $d \geq \bar{d}$, the exported amount of the initial endowment good is $(1-\alpha)\,Z$, which is, as we already pointed out earlier, the level of exports in a standard Walrasian economy. This is due to the fact that for $d \geq \bar{d}$ the level of appropriation is zero for both agents, which induces perfectly secure property rights (cf. equations (20), lower case). For $d < \bar{d}$, we observe the emergence of appropriation and with it insecure property rights, which induces a decrease in exports (cf. equations (20), upper case).

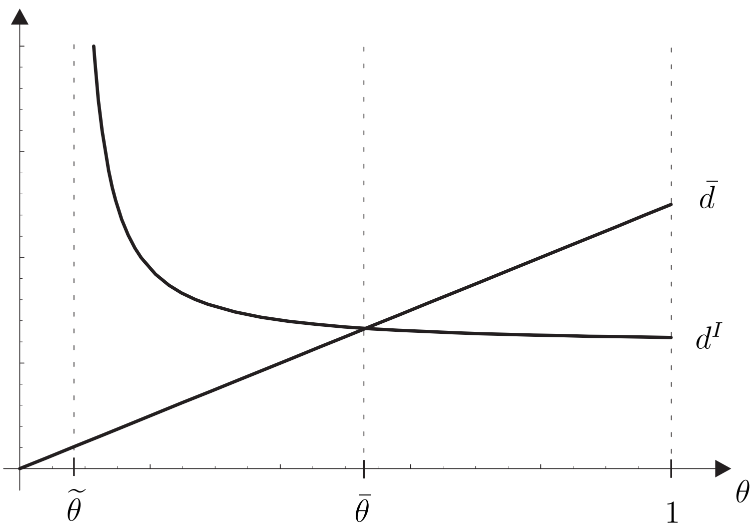

Figure 2: $d^*(\alpha, \theta)$ contingent on θ.

Consequently, the indirect utility function of agent i in the symmetric case becomes

$$v_i(d_i) = \alpha \ln[\alpha\, Z] + (1-\alpha)\ln[(1-\alpha)\, Z] - d_i, \tag{21}$$

for $d_i \geq \bar{d}$. Hence, $v_i(d_i)$ is a *decreasing* linear function of d_i, for $d_i \geq \bar{d}$. Accordingly, we know that $v_i(d_i)$ either has an interior maximum at a value of $d_i = d^I$ that satisfies

$$\left.\frac{\partial v_i}{\partial d_i}\right|_{d_i = d_j = d^I} = 0 \text{ with } d^I \in]0, \bar{d}\,], \tag{22}$$

or that v_i is maximized at a value of $d_i = \bar{d}$ that satisfies

$$\left.\frac{\partial v_i}{\partial d_i}\right|_{d_i = d_j = \bar{d}} \geq 0 \quad \text{with } \bar{d} \leq d^I. \tag{23}$$

At the border between the interior and corner solution the restriction on a ($a \geq 0$) is non binding ($d = d^I$, cf. equation (22)) and the level of defense is sufficient to deter appropriation ($d = \bar{d}$, cf. equation (23)). Thus, implementing $d = \bar{d}$ in equation (18) and solving this FOC for θ, delivers the case separating level of the exogenous effectiveness parameter:

$$\bar{\theta} = \frac{1}{4}\Big(2 + \alpha - 2\alpha^2\Big), \tag{24}$$

where it is easy to verify that $\bar{\theta} > \widetilde{\theta}$, $\forall\ \alpha \in]0, \hat{\alpha}[$, i.e., the case separating level of θ only emerges as long as there exists a solution to equation (18). Moreover, $\bar{\theta} \in [0, 1]$ $\forall\ \alpha \in]0, \hat{\alpha}[$.
We are able to show that $d^I \geq \bar{d}$ for $\theta \in]\widetilde{\theta}, \bar{\theta}]$ and $d^I < \bar{d}$ for $\theta \in]\bar{\theta}, 1]$ (see figure 2). Thus, as long as θ is sufficiently small ($\theta \in]0, \bar{\theta}]$), i.e. appropriation is sufficiently relative ineffective compared to defense, agents will implement a level of defense just sufficient to deter appropriation. For higher values of θ ($\theta \in]\bar{\theta}, 1]$), agents will choose a level of defense which lies below $\bar{d}$ and is thus insufficient to deter appropriation. Thus, conditions 22 and 23 imply that

$$d^*(\alpha, \theta) = \begin{cases} \bar{d} & \text{for } \theta \in \left]0, \bar{\theta}\right], \\ d^I < \bar{d} & \text{for } \theta \in \left]\bar{\theta}, 1\right], \end{cases} \tag{25}$$

which is graphically represented in figure 2.

We now turn to display the equilibrium values in the full game.

5 Equilibrium in the full game

Since we detected the emergence of different values of d^*, subject to the exogenous parameters α and θ in the first stage, we are now able to introduce the optimal level of a and z in the second stage subject to the mentioned parameters. As discussed, two different regimes emerge in equilibrium, contingent on the set of exogenous parameters α and θ:

$$\begin{aligned} \mathcal{A} &= \left\{(\alpha, \theta) \,\middle|\, 0 \leq \theta \leq \bar{\theta},\ 0 < \alpha < \hat{\alpha}\right\}, \\ \mathcal{B} &= \left\{(\alpha, \theta) \,\middle|\, \bar{\theta} < \theta \leq 1,\ 0 < \alpha < \hat{\alpha}\right\}. \end{aligned}$$

Thus, we are able to state the following proposition:

Proposition 3
(*Efficiency and security of property rights*)

1. *If* $(\alpha, \theta) \in \mathcal{A}$ *(first regime) the resulting equilibrium is* **non-aggressive**, *i.e.* $a^*(\alpha, \theta) = 0$, *which induces* **perfectly secure property rights** ($F_i^{i^*} = 1$). *Therefore, the resulting allocation is* **c-efficient**. *Inefficiencies arise due to the strictly positive level of defense, necessary to deter appropriation if* $\theta \neq 0$.

2. *If* $(\alpha, \theta) \in \mathcal{B}$ *(second regime) the resulting equilibrium is* **aggressive**, *i.e.* $a^*(\alpha, \theta) > 0$, *which induces* **insecure property rights** ($F_i^{i^*} < 1$). *Therefore, the resulting allocation is* **c-inefficient**: *Not all gains from trade are exhausted.*

The regime borders are graphically represented in figure 3. Contingent on these

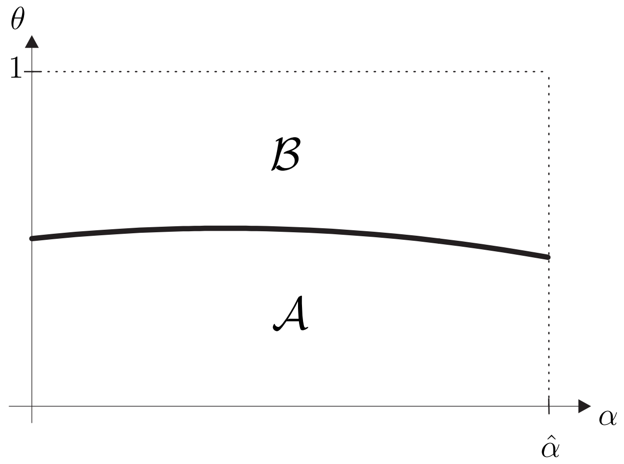

Figure 3: Regime–border

parameters, the level z, a, F_i^i and p result in equilibrium:

$$d^*(\alpha,\theta) = \begin{cases} \alpha\,\theta & \text{for } (\alpha,\theta) \in \mathcal{A}, \\ d^I & \text{for } (\alpha,\theta) \in \mathcal{B}, \end{cases} \tag{26a}$$

$$a^*(\alpha,\theta) = \begin{cases} 0 & \text{for } (\alpha,\theta) \in \mathcal{A}, \\ \frac{\sqrt{\alpha\,\theta\,d^I} - d^I}{\theta} & \text{for } (\alpha,\theta) \in \mathcal{B}, \end{cases} \tag{26b}$$

$$F_i^{i*}(\alpha,\theta) = \begin{cases} 1 & \text{for } (\alpha,\theta) \in \mathcal{A}, \\ \sqrt{\frac{d^I}{\alpha\,\theta}} & \text{for } (\alpha,\theta) \in \mathcal{B}, \end{cases} \tag{26c}$$

$$z^*(\alpha,\theta) = \begin{cases} \alpha\,Z & \text{for } (\alpha,\theta) \in \mathcal{A}, \\ \frac{\alpha\,Z\sqrt{\theta}}{\sqrt{\alpha\,d^I} + (1-\alpha)\sqrt{\theta}} & \text{for } (\alpha,\theta) \in \mathcal{B}, \end{cases} \tag{26d}$$

$$p^* = 1 \text{ for } (\alpha,\theta) \in \mathcal{A} \cup \mathcal{B}. \tag{26e}$$

Given that $(\alpha,\theta) \in \mathcal{A}$ the first regime emerges, which we introduced as the regime of *peaceful coexistence*. Here, appropriation is endogenously deterred (26b) by a sufficient investment in defensive activities in the first stage (26a), so that property rights of both agents are perfectly secure (26c). This induces c–efficiency, so that the endogenously determined market clearing price (26e) equals the marginal rate of substitution between both consumption goods for both agents in equilibrium. Both agents, given the initial endowment, export a fraction of $(1-\alpha)Z$ units of the domestic good $(Z - z^*(\alpha,\theta))$, which is equivalent to the traded amounts in a standard Walrasian economy (26d). Hence, all gains from trade are exhausted. It is worth noting that in this regime the resulting equilibria are not overall efficient, since there are remaining inefficiencies due to the investment of both agents necessary to deter appropriation, given that $\theta \neq 0$. For $\theta = 0$ the model considered here maps the standard textbook case of barter exchange. In a well-defined sense, then, the neoclassical structure is a special counterpart of the model considered here.
For $(\alpha,\theta) \in \mathcal{B}$, the second regime emerges, *trade and appropriation*. As has been said, in this regime the relative ineffectiveness of appropriation $(\theta > \bar{\theta})$ is insufficient to endogenously deter appropriation ((26a) and (26b)), which induces *insecure*

property rights (26c).[20] This has an unambiguous effect on trade volume (26d): Since the effective price of the domestic good is increased through the insecurity of property rights, both agents are now exporting less than in a standard Walrasian economy $(Z - z^*(\alpha, \theta))$.[21] Thus, the marginal rates of substitution between both consumption goods are unequal in equilibrium to the market clearing price, which represents the fact that there are unutilized gains from trade in equilibrium. This causes additional inefficiencies compared to the first regime: the resulting equilibrium is no longer c–efficient.
Thus, we were able to show that even if both consumptions goods are vital to the agents, non-aggressive equilibria emerge. The reason for this is that trade reduces the incentives to appropriate, which was our main argument.

6 Welfare aspects

At last, we take a closer look at the welfare aspects. For this, we need to specify the investment in defensive and appropriative activities in equilibrium. Since we are not able to explicitly display the level of defense contingent on α and θ in the second regime $(d^I(\alpha, \theta))$ we are forced to simulate d^I for various level of α. Given that $\alpha \in \left\{\frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}\right\}$ figure 5 - figure 10 on page 18 display the level of appropriation and defense in equilibrium $(a^*(\alpha, \theta),\, d^*(\alpha, \theta))$ contingent on θ. Moreover we see the sum of efforts of each agent in equilibrium in the contest $(e^*(\alpha, \theta))$, where

$$e^*(\alpha, \theta) = d^*(\alpha, \theta) + a^*(\alpha, \theta). \tag{27}$$

The key feature is now that the sum of efforts in equilibrium tends to rise, as θ rises. In the first regime $(\theta \leq \bar{\theta})$ the sum of efforts are equal to to the investment in defense, sufficient to deter appropriation. Therefore, the level of appropriation is zero. Moreover, as θ rises the investment in defense also rises.[22] As θ rises above $\bar{\theta}$ (second regime) there are two effects on the sum of efforts. First, the level of defense declines. Second, the level of appropriation rises. The combined effect on $e^*(\alpha, \theta)$, given that α is an element of the above set, is unambiguously positive. Therefore, $e^*(\alpha, \theta)$ is strictly higher in the second regime $(\theta > \bar{\theta})$ than in the first regime $(\theta \leq \bar{\theta})$. Given the results of the former chapter, we are now able to reason that any equilibrium in the second regime is *Pareto-dominated* by any equilibrium in the first regime, given that α is an element of the above set.
This fact is captured in figure 4: The upper curve represents the unconstrained Pareto payoff frontier, i.e. the one that would arise if claims to property were costlessly enforced $(\theta = 0)$ and efficient exchange could take place. Due to the symmetry assumptions, the resulting allocation in equilibrium is depicted by point (W), which represents an *overall efficient* allocation, since $a^* = d^* = 0$ and therefore no leisure time has been sacrificed $(l_i^* = L)$ in this case.
Given that $\theta \in]0, \bar{\theta}]$ (first regime sans the textbook case $(\theta = 0)$), the payoff frontier

[20] Since we already found out that $d^I < \bar{d} = \alpha\,\theta$ as long as $\theta > \bar{\theta}$, we see that $F_i^{i*} < 1$ in this regime.

[21] Again, this result stems from the fact that $d^I < \bar{d} = \alpha\,\theta$ for $\theta > \bar{\theta}$. Thus, the denominator of $z^*(\alpha, \theta)$ is always smaller than $\sqrt{\theta}$ and therefore $z^*(\alpha, \theta) > \alpha\, Z$ in the second regime.

[22] Recall that in this case $d^*(\alpha, \theta) = \bar{d} = \alpha\,\theta$

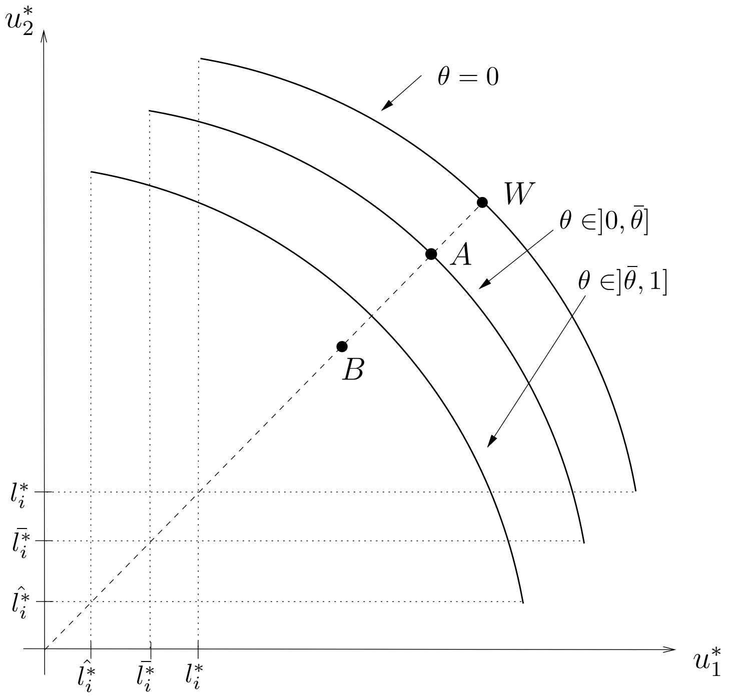

Figure 4: Pareto frontiers and equilibrium allocations of consumption goods contingent on θ

is *shifted inwards*. Since in this case the endogenously determined level of d is strictly positive in equilibrium ($d^*(\alpha, \theta) = \bar{d}$) the resulting level of leisure declines ($\bar{l}_i^* = L - e^*(\alpha, \theta) = L - \bar{d}$), compared to the unconstrained case. The resulting allocation in equilibrium (point A) is c-efficient, which is represented by the fact that A lies on the constrained Pareto frontier, but not *overall efficient* due to the positive investment in defense.
If $\theta > \bar{\theta}$ (second regime), the sum of efforts in the contest is strictly higher than in the first regime ($\hat{l}_i^* = L - e^*(\alpha, \theta) = L - (d^I + \frac{\sqrt{\alpha\,\theta\, d^I} - d^I}{\theta}) < \bar{l}_i^*$). Thus, the resulting restricted Pareto frontier in the second regime is represented by the inner curve. Point B now represents the resulting allocation in equilibrium in the second regime, which lies *off* the corresponding Pareto frontier. This captures the fact of *c-inefficiency*: Not all gains from trade are exhausted due to the insecure property rights.

7 Conclusion

The fact that property rights are perfectly secure and costlessly enforced is one major assumption for the rigorous neoclassical paradigm. From this starting point it is possible to explain the emergence of mutually advantageous trade. But this formulation of the economic problem is incomplete, since often property rights are not well defined or are costly to enforce. If this is true, the question arising is how the *shadow of conflict* reshapes the incentives to engage in mutually beneficial activities (as the trade of goods). Following recent literature we developed a two agent model which incorporates the potential of conflict in a standard neoclassical framework. This model emphasized the distinction between offensive weapons being the instruments of appropriation, and fortification, which provides defense against appropriation. The main goal was to show to what extent claims to property are ex-post secure, that is, after the possible trade of goods has occurred. We have shown that even in the absence of formal property rights, perfectly secure property rights can emerge in equilibrium, though (1) the contestable consumables are vital to the individuals and (2) initial endowments followed a binary distribution. The reason for this is that trading not only reduces the stock of goods subject to appropriation, but also reduces the propensity to engage in appropriation, due to the mutual beneficial effects of trade itself. In this regime (*peaceful coexistence*) the emerging trade of goods exhausts all gains from trade. However, inefficiencies arise due to the investment in defensive activities to deter appropriation. Thus, we have found an endogenously determined measure for the transaction costs underlying a market which replicates the outcome of a Walrasian market (zero appropriation and efficient exchange).

These findings only apply, if the relative effectiveness of appropriation compared to defending initial claims is sufficiently small. If appropriation is not sufficiently ineffective compared to fortification, trade and appropriation emerges in equilibrium (second regime). Since claims to property are not fully secure incentives to trade are violated. This has an unambiguously effect on the trade volume - it declines. In this regime there are unutilized gains from trade in equilibrium.

Moreover, we were able to show that every equilibrium in the first regime strictly Pareto-dominates every equilibrium in the second regime. This is due to two facts: First, the first regime, unlike the second regime, exhibits c-efficiency. Secondly, the efforts in the contest, i.e. the sum of investments in defensive and appropriative activities, is strictly higher in the second regime.

Summing things up, we showed that in contrast to the aforementioned literature, efficient market-based exchange of goods is possible, although property rights are not exogenuously and costlessly enforced, although appropriation is not destructive and although the circumstances (preferences and initial endowments) are awkward.

Simulation

The value of appropriation ($a^*(\alpha,\theta)$), defense ($d^*(\alpha,\theta)$) and the sum of efforts ($e^*(\alpha,\theta)$) in equilibrium contingent on θ for various levels of α.

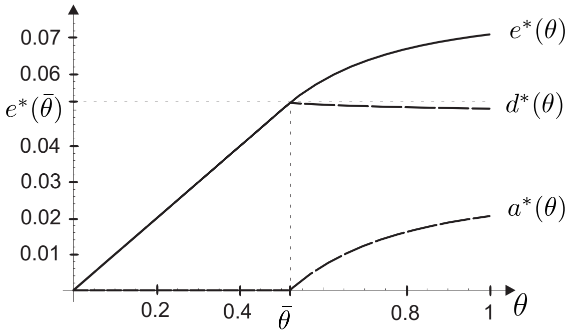

Figure 5: $\alpha = \frac{1}{10}$.

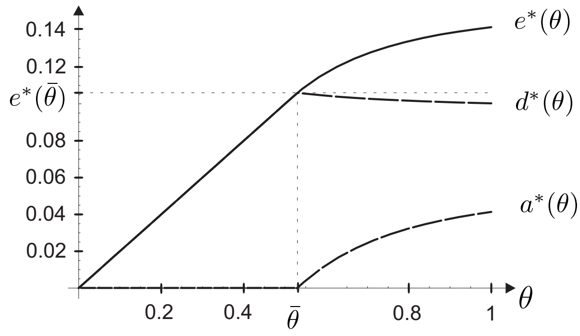

Figure 6: $\alpha = \frac{1}{5}$.

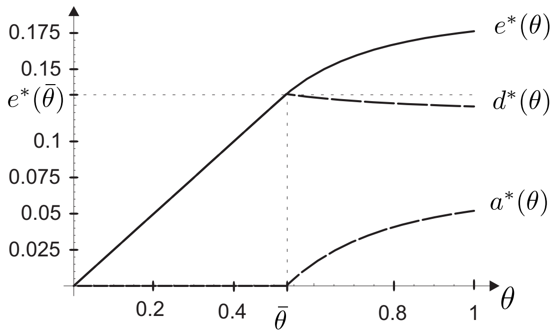

Figure 7: $\alpha = \frac{1}{4}$.

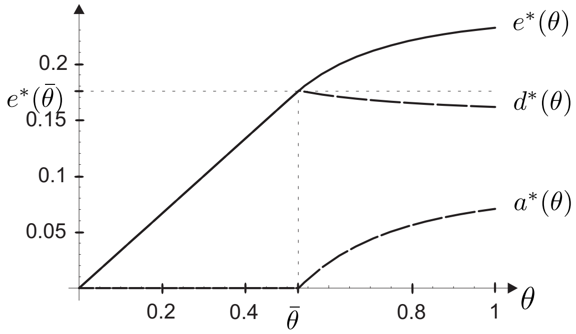

Figure 8: $\alpha = \frac{1}{3}$.

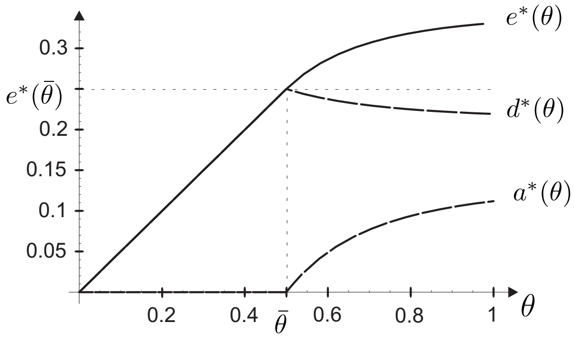

Figure 9: $\alpha = \frac{1}{2}$.

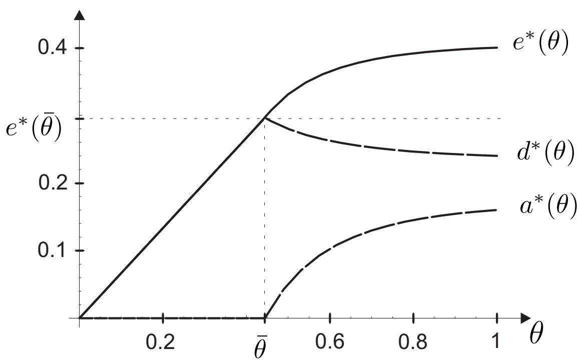

Figure 10: $\alpha = \frac{3}{5}$.

Appendix (not for publication)

A Stage 2

The utility maximizing problem for agent i in stage 2 becomes:

$$\max_{a_i, z_i} \quad u_i(c_i^i, c_i^j, l_i)$$
$$\text{s.t.} \quad a_i \geq 0, \quad z_i \geq 0, \quad z_i \leq Z^i$$

The Lagrangian becomes:

$$L_i(a_i, z_i) = u_i(c_i^i, c_i^j, l_i) + \mu_i\,(Z^i - z_i).$$

Partial derivation leads to

$$\begin{aligned}
\frac{\partial L_i}{\partial a_i} &\leq 0, \qquad & a_i \geq 0 \qquad & \text{and} \qquad & \frac{\partial L_i}{\partial a_i}\,a_i = 0,\\
\frac{\partial L_i}{\partial z_i} &\leq 0, \qquad & z_i \geq 0 \qquad & \text{and} \qquad & \frac{\partial L_i}{\partial z_i}\,z_i = 0,\\
\frac{\partial L_i}{\partial \mu_i} &\geq 0, \qquad & \mu_i \geq 0 \qquad & \text{and} \qquad & \frac{\partial L_i}{\partial \mu_i}\,\mu_i = 0,
\end{aligned}$$

where each third term represents the complementary slackness condition.
Given the quasi–linear preferences (equation 13, page 7), the utility function for agent 1 becomes:

$$\begin{aligned}
u_1 =&\alpha \ln\Big[F_1^1(d_1, a_2, \theta)\,z_1\Big] + (1-\alpha)\ln\left[F_1^2(d_2, a_1, \theta)\,z_2 + \frac{(Z^1 - z_1)p^1}{p^2}\right]\\
&+ L - a_1 - d_1,
\end{aligned} \tag{28.1}$$

and for agent 2:

$$\begin{aligned}
u_2 =&\alpha \ln\Big[F_2^2(d_2, a_1, \theta)\,z_2\Big] + (1-\alpha)\ln\left[F_2^1(d_1, a_2, \theta)\,z_1 + \frac{(Z^2 - z_2)p^2}{p^1}\right]\\
&+ L - a_2 - d_2,
\end{aligned} \tag{28.2}$$

where $(28.i)$ represents a function of the ith agent.
Implementing κ_i and λ_i as shadow price for violating the non-negativity constraints on a_i and z_i, respectively, we acquire the following modified Lagrangian:

$$\mathcal{L}_1(a_1, z_1) = u_1 + \kappa_1\,a_1 + \lambda_1\,z_1 + \mu_1(Z - z_1) \tag{29.1}$$

and

$$\mathcal{L}_2(a_2, z_2) = u_2 + \kappa_2\,a_2 + \lambda_2\,z_2 + \mu_2(Z - z_2). \tag{29.2}$$

Given this functional specification the FOCs (equation 10 and 11, page 6) yield:

$$\frac{\partial \mathcal{L}_1}{\partial a_1} = \frac{(1-\alpha)\,d_2\,z_2\,p^2\,\theta}{(d_2 + a_1\theta)\Big(p^1(d_2 + a_1\,\theta)(Z^1 - z_1) + a_1\,\theta\,z_2\,p^2\Big)} - 1 + \kappa_1 \overset{!}{=} 0, \tag{30.1}$$

$$\frac{\partial \mathcal{L}_2}{\partial a_2} = \frac{(1-\alpha)\, d_1\, z_1\, p^1\, \theta}{(d_1 + a_2\,\theta)\Big(p^2(d_1 + a_2\,\theta)(Z^2 - z_2) + a_2\,\theta\, z_1\, p^1\Big)} - 1 + \kappa_2 \overset{!}{=} 0, \tag{30.2}$$

$$\frac{\partial \mathcal{L}_1}{\partial z_1} = \frac{\alpha}{z_1} - \frac{(1-\alpha)\,(a_1\,\theta + d_2)p^1}{a_1\,\theta\, z_2\, p^2 + (Z^1 - z_1)\,(a_1\theta + d_2)p^1} + \lambda_1 - \mu_1 \overset{!}{=} 0, \tag{31.1}$$

$$\frac{\partial \mathcal{L}_2}{\partial z_2} = \frac{\alpha}{z_2} - \frac{(1-\alpha)\,(a_2\,\theta + d_1)\, p^2}{a_2\,\theta\, z_1\, p^1 + \,(Z^2 - z_2)\,(a_2\theta + d_1)p^2} + \lambda_2 - \mu_2 \overset{!}{=} 0. \tag{31.2}$$

Setting $Z^1 = Z^2 = Z$, $p^1 = 1$ and $p^2 = 2$, we are now able to distinguish between the following six cases (since $z_i = 0$ and $Z - z_i = 0$ can not emerge simultaneously, there are only six, rather than eight possible cases).

Case	a_i	κ_i	z_i	λ_i	$Z - z_i$	μ_i
1.	≥ 0	$= 0$	≥ 0	$= 0$	≥ 0	$= 0$
2.	$= 0$	≥ 0	≥ 0	$= 0$	≥ 0	$= 0$
3.	≥ 0	$= 0$	≥ 0	$= 0$	$= 0$	≥ 0
4.	$= 0$	≥ 0	≥ 0	$= 0$	$= 0$	≥ 0
5.	≥ 0	$= 0$	$= 0$	≥ 0	≥ 0	$= 0$
6.	$= 0$	≥ 0	$= 0$	≥ 0	≥ 0	$= 0$

A.1 First case: Interior solution

Given that none of the restrictions are binding in equilibrium, it follows that

$$\begin{aligned} a_i \geq 0 \quad &\Longrightarrow \quad \kappa_i = 0, \\ z_i \geq 0 \quad &\Longrightarrow \quad \lambda_i = 0, \\ Z - z_i \geq 0 \quad &\Longrightarrow \quad \mu_i = 0. \end{aligned}$$

Therefore the following reaction–functions result:

$$z_1(z_2, d_2, p, a_1) = \alpha\,\big(Z + F_1^2(d_2, a_1)\, p\, z_2\big), \tag{32.1}$$

$$z_2(z_1, d_1, p, a_2) = \alpha\left(Z + \frac{F_2^1(d_1, a_2)z_1}{p}\right). \tag{32.2}$$

In order to determine the equilibrium values, we have to solve the partial derivations (equations 30 and 31) simultaneously while taking into account, that in equilibrium,

$$\frac{Z - z_1(d_2, a_1, p, z_2)}{p} \overset{!}{=} Z - z_2(d_1, a_2, p, z_1). \tag{33}$$

This delivers the following equilibrium values:

$$a_i(d_i, d_j) = \frac{(-2+\alpha)\, d_j - \alpha\,\sqrt{d_i\, d_j} + \sqrt{\alpha\, d_j}\,\sqrt{\alpha\,\left(\sqrt{d_i} - \sqrt{d_j}\right)^2 + 4\theta}}{2\,\theta}, \tag{34a}$$

$$F_i^i(d_i, d_j) = \frac{2d_i}{\alpha(d_i - \sqrt{d_i\, d_j}) + \sqrt{\alpha\, d_i\,(\alpha(d_i - d_j)^2 + 4\,\theta)}}, \tag{34b}$$

$$z_i(d_i, d_j) = \frac{\big(-1 + \alpha\, F_j^j(d_i, d_j)\big)\, \alpha\, Z}{-1 + \alpha - \alpha^2\, F_i^j(d_i, d_j)\, F_i^i(d_i, d_j)}, \tag{34c}$$

$$p\,(d_1, d_2) = \frac{-1 + \alpha - \alpha^2\, F_2^2(d_2, d_1)\, F_2^1(d_1, d_2)}{-1 + \alpha - \alpha^2\, F_1^1(d_1, d_2)\, F_1^2(d_2, d_1)}, \tag{34d}$$

for $d_i, d_j > 0$, with $i \neq j$.

A.2 Second case: First corner solution

In the first corner solution, both individuals invest no time in appropriation ($a_i = 0$) and the restriction on z_i ($z_i \in [0, Z]$) is not binding:

$$\begin{aligned} a_i = 0 \quad &\Longrightarrow \quad \kappa_i \geq 0, \\ z_i \geq 0 \quad &\Longrightarrow \quad \lambda_i = 0, \\ Z \geq z_i \quad &\Longrightarrow \quad \mu_i = 0, \end{aligned}$$

The first order conditions thus become:

$$\frac{(1-\alpha)\, z_2\, d_2\, p\, \theta}{d_2^2\,(Z - z_1)} - 1 + \kappa_1 \overset{!}{=} 0, \tag{35.1}$$

$$\frac{(1-\alpha)\, d_1\, \theta\, z_1}{d_1^2\, p\,(Z - z_2)} - 1 + \kappa_2 \overset{!}{=} 0, \tag{35.2}$$

$$\frac{\alpha}{z_1} - \frac{1-a}{Z - z_1} \overset{!}{=} 0, \tag{36.1}$$

$$\frac{\alpha}{z_2} - \frac{1-\alpha}{Z - z_2} \overset{!}{=} 0. \tag{36.2}$$

Solving equations 36 while taking into account, that

$$\frac{Z - z_1}{p} \overset{!}{=} Z - z_2 \tag{37}$$

in equilibrium, delivers $z_1 = \alpha Z$, $z_2 = \alpha Z$ and $p = 1$. Applying these values into equations 35 delivers

$$\kappa_1 = 1 - \frac{\alpha\, \theta}{d_2}, \tag{38.1}$$

$$\kappa_2 = 1 - \frac{\alpha\, \theta}{d_1}. \tag{38.2}$$

Since we know that in the corner solution $\kappa_i \geq 0$ this delivers:

$$d_i \geq \alpha\, \theta \tag{39}$$

in a corner solution. Summing up, we find that:

$$a_i = 0, \tag{40a}$$

$$F_i^i = 1, \tag{40b}$$

$$z_i = \alpha Z, \tag{40c}$$

$$p = 1, \tag{40d}$$

for $d_i \geq \alpha\, \theta$.

A.3 Third case: Second corner solution

In the third case $z_i = Z$ and the restriction on a_i ($a_i \geq 0$) is not binding.

$$\begin{aligned} a_i \geq 0 \quad &\Longrightarrow \quad \kappa_i = 0, \\ z_i \geq 0 \quad &\Longrightarrow \quad \lambda_i = 0, \\ z_i = Z \quad &\Longrightarrow \quad \mu_i \geq 0. \end{aligned}$$

The FOCs thus become:

$$\frac{(1-\alpha)\, d_2}{a_1(d_2 + a_1\,\theta)} - 1 \overset{!}{=} 0, \tag{41.1}$$

$$\frac{(1-\alpha)\, d_1}{a_2(d_1 + a_2\,\theta)} - 1 \overset{!}{=} 0, \tag{41.2}$$

$$-\mu_1 + \frac{\alpha}{Z} - \frac{(1-\alpha)(d_1 + a_2\,\theta)}{a_2\,\theta\, Z} \overset{!}{=} 0, \tag{42.1}$$

$$-\mu_2 + \frac{\alpha}{Z} - \frac{(1-\alpha)(d_2 + a_1\,\theta)}{a_1\,\theta\, Z} \overset{!}{=} 0, \tag{42.2}$$

Solving equations 41 for a_1 and a_2, respectively delivers:

$$a_1 = \frac{-d_2 + \sqrt{d_2(d_2 + 4(1-\alpha)\,\theta)}}{2\,\theta} \tag{43.1}$$

$$a_2 = \frac{-d_1 + \sqrt{d_1(d_1 + 4(1-\alpha)\,\theta)}}{2\,\theta} \tag{43.2}$$

Implementing equations 43 into equations 42 and solving for the shadow prices, delivers

$$\mu_1 = -\frac{d_2 + (2 - 4\,\alpha)\,\theta + \sqrt{d_2\big(d_2 + 4\,\theta(1-\alpha)\big)}}{2\,\theta\, Z}, \tag{44.1}$$

$$\mu_2 = -\frac{d_1 + (2 - 4\,\alpha)\,\theta + \sqrt{d_1\big(d_1 + 4\,\theta(1-\alpha)\big)}}{2\,\theta\, Z}. \tag{44.2}$$

Since we know that in this corner solution $\mu_i \geq 0$, we find that the following conditions have to be satisfied

$$d_j \leq \frac{(1-2\,\alpha)^2\,\theta}{\alpha} \quad \wedge \quad \alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[. \tag{45}$$

Summing up, in the second corner solution, we find in equilibrium that

$$a_i = \frac{-d_j + \sqrt{d_j(d_j + 4(1-\alpha)\,\theta)}}{2\theta}, \tag{46a}$$

$$F_i^i = \frac{2\,d_i}{d_i + \sqrt{d_i\big(d_i + 4(1-\alpha)\,\theta\big)}}, \tag{46b}$$

$$z_i = Z, \tag{46c}$$

for $d_j \leq \frac{(1-2\,\alpha)^2\,\theta}{\alpha}$ and $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$.

A.4 Forth case: Third corner solution

In the forth case $a_i = 0$ and $z_i = Z$:

$$\begin{aligned} a_i = 0 \quad &\Longrightarrow \quad \kappa_i \geq 0, \\ z_i \geq 0 \quad &\Longrightarrow \quad \lambda_i = 0, \\ z_i = Z \quad &\Longrightarrow \quad \mu_i \geq 0. \end{aligned}$$

In this case the denominator of the fraction in equations 30 will go to zero, thus the value of these equations will go to infinity. Since neither consumption goods are traded ($z_i = Z$), nor appropriated ($a_i = 0$), the equilibrium consumption would for each agent consist of none foreign goods. Since, $\frac{\partial u_i(\cdot)}{\partial c_i^j}\Big|_{c_i^j \to 0} \to \infty$. (see equation (12), page 6), this is no equilibrium.

A.5 Fifth case

In the fifth case $z_i = 0$ and the restriction on a_i ($a_i \geq 0$) is not binding.

$$\begin{aligned} a_i \geq 0 \quad &\Longrightarrow \quad \kappa_i = 0, \\ z_i = 0 \quad &\Longrightarrow \quad \lambda_i \geq 0, \\ Z \geq 0 \quad &\Longrightarrow \quad \mu_i \geq 0. \end{aligned}$$

In this case the values of equations 31 will go to infinity, i.e. $z_i = 0$ is never an equilibrium.

A.6 Sixth case

In the sixth case $z_i = 0$ and $a_i = 0$.

$$\begin{aligned} a_i = 0 \quad &\Longrightarrow \quad \kappa_i \geq 0, \\ z_i = 0 \quad &\Longrightarrow \quad \lambda_i \geq 0, \\ Z \geq z_i \quad &\Longrightarrow \quad \mu_i \geq 0. \end{aligned}$$

Again, this case needs no further consideration, since $z_i = 0$ never can emerge in equilibrium.

B Offer–curve, for $F_i^i \in [0,1]$

For the following explanation we set $i \neq j$. Given an appointed and identical investment in appropriation (a_i) and defense (d_i) for both agents, it follows an appointed and identical defended fraction (F_i^i) for both agents. If we denote the amount of non–exported goods of agent j with z_j, the amount of non–exported goods of agent i with z_i and the amount of imported goods of agent i with $\tilde{z}_i$, we get the following utility function:

$$u_i = \alpha \ln[F_i^i \, z_i] + (1 - \alpha) \ln[F_i^j \, z_j + \tilde{z}_i] + L - a_i - d_i, \tag{47}$$

with $i \neq j$. The budget constraint is then

$$p^i\, Z^i - p^i z_i - p^j\, \tilde{z}_i. \geq 0. \tag{48}$$

If each agent maximizes his utility by choosing z_i and $\tilde{z}_i$ under given restrictions

$$\max_{z_i, \tilde{z}_i} \qquad \alpha \ln[F_i^i\, z_i] + (1-\alpha) \ln[F_i^j\, z_j + \tilde{z}_i] + L - a_i - d_i \tag{49}$$

$$\text{s.t.} \qquad p^i\, Z^i - p^i z_i - p^j\, \tilde{z}_i \geq 0, \tag{50}$$

we get the following Marshallian demand functions

$$z_i(p^i, p^j) \;=\; \alpha\, Z^i + \frac{\alpha\, F_i^j\, p^j\, z_j}{p^i}, \tag{51}$$

$$\tilde{z}_i(p^i, p^j) \;=\; \frac{(1-\alpha) Z^i\, p^i}{p^j} - \alpha\, F_i^j\, z_j, \tag{52}$$

where $z_i(p^i, p^j)$ represents the demand for the domestic consumption good and $\tilde{z}_i(p^i, p^j)$ represents the demand for the foreign good. For $F_i^j = 0$ (perfectly secure property rights) these Marshallian functions equal the one in a standard Walrasian economy.

In equilibrium the non-exported goods of agent j (z_j) have to equal the initial–endowment of agent j (Z^j) minus the demand for foreign goods of agent i, that is

$$z_j = Z^j - \tilde{z}_i. \tag{53}$$

Thus, the equilibrium value of the Marshallian demand functions become:

$$z_i^*(p^i, p^j) \;=\; \frac{\alpha\, Z^i p^i + \alpha\, F_i^j (p^j\, Z^j - p^i\, Z^i)}{p^i (1 - \alpha\, F_i^j)}, \tag{54}$$

$$\tilde{z}_i^*(p^i, p^j) \;=\; \frac{(1-a)\, p^i\, Z^i - \alpha\, F_i^j\, p^j\, Z^j}{p^j (1 - \alpha\, F_i^j)}. \tag{55}$$

If we set $p^i = 1$ we can now determine the offer–curve for agent i:

$$OC_i(\tilde{z}_i) = \frac{\alpha\, Z^i (F_j^j\, \tilde{z}_i + F_i^j\, Z^j)}{\tilde{z}_i + \alpha\, F_i^j (Z^j - \tilde{z}_i)}, \tag{56}$$

which is a continuous, declining and convex function of $\tilde{z}_i$, for $\tilde{z}_i \in [0, Z^j]$, $\alpha \in]0, \hat{\alpha}[$ and $F_i^i \in [0, 1[$:

$$\frac{\partial OC_i}{\partial \tilde{z}_i} = -\frac{(1-\alpha)\alpha F_i^j Z^i Z^j}{(\alpha\, F_i^j (Z^j - \tilde{z}_i) + \tilde{z}_i)^2} < 0, \tag{57}$$

$$\frac{\partial^2 OC_i}{\partial \tilde{z}_i^2} = \frac{2(1-\alpha)\alpha F_i^j (1 - \alpha F_i^j) Z^i Z^j}{(\alpha F_i^j (Z^j - \tilde{z}_i) + \tilde{z}_i)^3} > 0. \tag{58}$$

For $F_i^i(\cdot) = 1$ the offer-curve takes the typical form:

$$OC_i = \alpha\, Z^i. \tag{59}$$

C The FOCs in the first stage

The utility maximizing problem for agent i in stage 1 becomes:

$$\begin{array}{cc} \max\limits_{d_i} & v_i(d_i, d_j) \\ \text{s.t.} & d_i \geq 0. \end{array}$$

Assuming that the restriction on a_i and z_i is not binding, the Lagrangian becomes

$$\mathcal{L}_i = v_i(d_i, d_j) + \eta_i\, d_i, \tag{60}$$

where η_i represents the shadow price for violating the non-negativity constraint on d_i and $v_i(d_i, d_j)$ is represented by (17.1) for agent 1 and (17.2) for agent 2 (page 10). Using the envelope theorem and assuming that $\eta_i = 0$, this yields the following FOCs:

$$\begin{aligned} &\frac{\alpha}{F_1^1(d_1,d_2)}\,\frac{a_2(d_1,d_2) - d_1\theta\frac{\partial a_2(d_1,d_2)}{\partial d_1}}{\big(\theta a_2(d_1,d_2) + d_1\big)^2} \\ &\qquad + \frac{(1-\alpha)}{F_1^2(d_1,d_2)\, z_2(d_1,d_2) + \underbrace{\dfrac{Z - z_1(d_1,d_2)}{p(d_1,d_2)}}_{\Phi_1}} \\ &\qquad \left[F_1^2(d_1,d_2)\,\frac{\partial z_2(d_1,d_2)}{\partial d_1} \underbrace{- \frac{Z - z_1(d_1,d_2)}{\big(p(d_1,d_2)\big)^2}\,\frac{\partial p(d_1,d_2)}{\partial d_1}}_{\Psi_1}\right] - 1 \stackrel{!}{=} 0, \end{aligned} \tag{61.1}$$

and for agent 2:

$$\begin{aligned} &\frac{\alpha}{F_2^2(d_1,d_2)}\,\frac{a_1(d_1,d_2) - d_2\theta\frac{\partial a_1(d_1,d_2)}{\partial d_2}}{\big(\theta\, a_1(d_1,d_2) + d_2\big)^2} \\ &+ \frac{(1-\alpha)}{F_2^1(d_1,d_2)\, z_1(d_1,d_2) + \underbrace{\big(Z - z_2(d_1,d_2)\big)\, p\,(d_1,d_2)}_{\Phi_2}} \\ &\qquad \left[F_2^1(d_1,d_2)\,\frac{\partial z_1(d_1,d_2)}{\partial d_2} + \underbrace{\big(Z - z_2(d_1,d_2)\big)\,\frac{\partial p(d_1,d_2)}{\partial d_2}}_{\Psi_2}\right] - 1 \stackrel{!}{=} 0. \end{aligned} \tag{61.2}$$

It is obvious that – except for terms Φ_i and Ψ_i – the FOCs are symmetric for both agents. We can proof that even these two terms are symmetric:
At first we will show that Φ_1 is symmetric to Φ_2. Given the equilibrium values out of the second stage, the value of imports is for agent 1

$$\begin{aligned} \Phi_1 &= \frac{Z - z_1(d_1,d_2)}{p(d_1,d_2)} \\ \Longleftrightarrow \Phi_1 &= \frac{(2\,\alpha - 1)Z - \alpha^2\, Z(F_1^2\, F_1^1 + F_2^2)}{-1 + \alpha - \alpha^2\, F_2^2\, F_2^1} \end{aligned} \tag{62}$$

and for agent 2

$$
\begin{aligned}
\Phi_2 =& \frac{Z - z_2(d_2, d_1)}{p(d_2, d_1)} \\
\Longleftrightarrow \Phi_2 =& \frac{(2\,\alpha - 1)Z - \alpha^2\, Z(F_2^1\, F_2^2 + F_1^1)}{-1 + \alpha - \alpha^2\, F_1^1\, F_1^2}.
\end{aligned}
\tag{63}
$$

As we can now easily see, Φ_1 is symmetric to Φ_2.
Next we proof that Ψ_1 is symmetric to Ψ_2:

$$
\Psi_1 \;=\; -\frac{Z - z_1(d_1, d_2)}{\big(p(\cdot)\big)^2}\,\frac{\partial p(\cdot)}{\partial d_1}, \tag{64}
$$

$$
\Psi_2 \;=\; \big(Z - z_2(d_1, d_2)\big)\,\frac{\partial p(\cdot)}{\partial d_2}. \tag{65}
$$

The partial derivation of $p(d_1, d_2)$ subject to d_1 is:

$$
\begin{aligned}
\frac{\partial p(\cdot)}{\partial d_1} = \Bigg(&- \alpha^2 \left(\frac{\partial F_2^2}{\partial a_1}\frac{\partial a_1}{\partial d_1} F_2^1 + F_2^2 \left(\frac{\partial F_2^1}{\partial d_1} + \frac{\partial F_2^1}{\partial a_2}\frac{\partial a_2}{\partial d_1}\right)\right)\Bigg) \\
&\Big(- 1 + \alpha - \alpha^2 F_1^1 F_1^2\Big) + \alpha^2 \left(\left(\frac{\partial F_1^1}{\partial d_1} + \frac{\partial F_1^1}{\partial a_2}\frac{\partial a_2}{\partial d_1}\right) F_1^2 + F_1^1 \frac{\partial F_1^2}{\partial a_1}\frac{\partial a_1}{\partial d_1}\right) \\
&\Big(- 1 + \alpha - \alpha^2 F_2^2 F_2^1\Big)\Bigg) \Bigg/ \Big(- 1 + \alpha - \alpha^2 F_1^1 F_1^2\Big)^2
\end{aligned}
\tag{64a}
$$

and

$$
-\frac{Z - z_1(\cdot)}{\big(p(\cdot)\big)^2} = -\frac{\Big(\alpha^2\, F_1^2\, F_2^1 - (1-\alpha)^2\Big) Z}{-1 + \alpha - \alpha^2 F_1^2 F_1^1}\, \frac{\Big(- 1 + \alpha - \alpha^2 F_1^1 F_1^2\Big)^2}{\Big(- 1 + \alpha - \alpha^2 F_2^2 F_2^1\Big)^2}. \tag{64b}
$$

So that

$$
\begin{aligned}
\Psi_1 = \quad & \Big(\alpha^2 F_1^2 F_2^1 - (1-a)^2\Big) Z \\
& \Bigg(\alpha^2 \bigg(\frac{\partial F_2^2}{\partial a_1}\frac{\partial a_1}{\partial d_1} F_2^1 + F_2^2 \left(\frac{\partial F_2^1}{\partial d_1} + \frac{\partial F_2^1}{\partial a_2}\frac{\partial a_2}{\partial d_1}\right)\bigg)\Big(- 1 + \alpha - \alpha^2 F_1^1 F_1^2\Big) \\
& - \alpha^2 \left(\left(\frac{\partial F_1^1}{\partial d_1} + \frac{\partial F_1^1}{\partial a_2}\frac{\partial a_2}{\partial d_1}\right) F_1^2 + F_1^1 \frac{\partial F_1^2}{\partial a_1}\frac{\partial a_1}{\partial d_1}\right) \Big(- 1 + \alpha - \alpha^2 F_2^2 F_2^1\Big)\Bigg) \\
& \Bigg/ \left(\Big(- 1 + \alpha - \alpha^2 F_2^2 F_2^1\Big)^2 \Big(- 1 + \alpha - \alpha^2 F_1^2 F_1^1\Big)\right).
\end{aligned}
\tag{64'}
$$

The partial derivation of $p(d_1, d_2)$ subject to d_2 is

$$\frac{\partial p(\cdot)}{\partial d_2} = \Bigg(-\alpha^2 \left(\left(\frac{\partial F_2^2}{\partial d_2} + \frac{\partial F_2^2}{\partial a_1}\frac{\partial a_1}{\partial d_2} \right) F_2^1 + F_2^2 \frac{\partial F_2^1}{\partial a_2}\frac{\partial a_2}{\partial d_2} \right)$$
$$\Big(-1+\alpha-\alpha^2 F_1^1 F_1^2 \Big) + \alpha^2 \left(\frac{\partial F_1^1}{\partial a_2}\frac{\partial a_2}{\partial d_2} F_1^2 + F_1^1 \left(\frac{\partial F_1^2}{\partial d_2} + \frac{\partial F_1^2}{\partial a_1}\frac{\partial a_1}{\partial d_2} \right) \right) \tag{65a}$$
$$\Big(-1+\alpha-\alpha^2 F_2^2 F_2^1 \Big) \Bigg) \Bigg/ \Big(-1+\alpha-\alpha^2 F_1^1 F_1^2 \Big)^2$$

and

$$Z - z_2(\cdot) = \frac{\Big(\alpha^2\, F_2^1\, F_1^2 - (1-\alpha)^2\Big) Z}{-1+\alpha-\alpha^2 F_2^1 F_2^2}. \tag{65b}$$

That is

$$\Psi_2 = \Big(\alpha^2\, F_2^1\, F_1^2 - (1-\alpha)^2\Big) Z$$
$$\Bigg(\alpha^2 \left(\frac{\partial F_1^1}{\partial a_2}\frac{\partial a_2}{\partial d_2} F_1^2 + F_1^1 \left(\frac{\partial F_1^2}{\partial d_2} + \frac{\partial F_1^2}{\partial a_1}\frac{\partial a_1}{\partial d_2} \right) \right) \Big(-1+\alpha-\alpha^2 F_2^2 F_2^1 \Big)$$
$$-\alpha^2 \left(\left(\frac{\partial F_2^2}{\partial d_2} + \frac{\partial F_2^2}{\partial a_1}\frac{\partial a_1}{\partial d_2} \right) F_2^1 + F_2^2 \frac{\partial F_2^1}{\partial a_2}\frac{\partial a_2}{\partial d_2} \right) \Big(-1+\alpha-\alpha^2 F_1^1 F_1^2 \Big) \Bigg)$$
$$\Bigg/ \Big(-1+\alpha-\alpha^2 F_1^1 F_1^2 \Big)^2 \Big(-1+\alpha-\alpha^2 F_2^1 F_2^2 \Big). \tag{65'}$$

Thus, we see that both terms are symmetric. That is,

$$\Psi_i = \Big(\alpha^2 F_i^j F_j^i - (1-a)^2\Big) Z$$
$$\Bigg(\alpha^2 \Big(\frac{\partial F_j^j}{\partial a_i}\frac{\partial a_i}{\partial d_i} F_j^i + F_j^j \left(\frac{\partial F_j^i}{\partial d_i} + \frac{\partial F_j^i}{\partial a_j}\frac{\partial a_j}{\partial d_i} \right) \Big) \Big(-1+\alpha-\alpha^2 F_i^i F_i^j \Big)$$
$$-\alpha^2 \left(\left(\frac{\partial F_i^i}{\partial d_i} + \frac{\partial F_i^i}{\partial a_j}\frac{\partial a_j}{\partial d_i} \right) F_i^j + F_i^i \frac{\partial F_i^j}{\partial a_i}\frac{\partial a_i}{\partial d_i} \right) \Big(-1+\alpha-\alpha^2 F_j^j F_j^i \Big) \Bigg) \tag{66}$$
$$\Bigg/ \Bigg(\Big(-1+\alpha-\alpha^2 F_j^j F_j^i \Big)^2 \Big(-1+\alpha-\alpha^2 F_i^j F_i^i \Big) \Bigg),$$

with $i \neq j$.
Since we know that the FOCs of both agents in the first stage are symmetric, we know that $d_1 = d_2 = d$ is a Nash-equilibrium. Without loss of generality we will take a look at the solution of equation (61.1), i.e. the solution to the FOC in stage one for agent 1, under the assumption that $d_1 = d_2 = d$. Using this symmetry property

we find that

$$
\begin{aligned}
a(d) &= a_i(d_i,d_j)\Big|_{d_i=d_j=d} = \frac{\sqrt{\alpha\,\theta\,d}-d}{\theta}, && \text{(67a)}\\
F_i^i(d) &= F_i^i(d_j,d_j)\Big|_{d_i=d_j=d} = \sqrt{\frac{d}{\alpha\,\theta}}, && \text{(67b)}\\
z(d) &= z_i(d_i,d_j)\Big|_{d_i=d_j=d} = \frac{\alpha\sqrt{\theta}Z}{\sqrt{\alpha\,d}+(1-\alpha)\sqrt{\theta}}, && \text{(67c)}\\
p &= p(d_1,d_2)\Big|_{d_1=d_2=d} = 1, && \text{(67d)}
\end{aligned}
$$

which correspond to equations (20) on page 11. Thus, the FOC in the first stage (equation (61.1)) yields in the symmetric case:

$$
\frac{\alpha\,\theta}{F_1^1(d)}\left(\frac{a(d)-\frac{\partial a_2}{\partial d_1}\Big|_{d_1=d_2=d}}{(d_1+\theta\,a_2)^2}\right)+\frac{1-\alpha}{Z-F_1^1(d)\,z(d)}\left(\left(1-F_1^1(d)\right)\frac{\partial z_2}{\partial d_1}\Big|_{d_1=d_2=d}\right.
$$
$$
\left.-\left(Z-z(d)\right)\frac{\partial p(d_1,d_2)}{\partial d_1}\Big|_{d_1=d_2=d}\right)=0,
$$

which, after some tedious algebra, leads to

$$
\begin{aligned}
H(\alpha,\theta,d)=&-1+\frac{\alpha\left(2-\sqrt{\frac{\alpha\,d}{\theta}}\right)}{4\,d}\\
&+\frac{\alpha^2\left(1-\alpha+\sqrt{\frac{\alpha\,d}{\theta}}\right)\left(d\left(2-\alpha+\sqrt{\frac{\alpha\,d}{\theta}}\right)-2\,\alpha\sqrt{\alpha\,d\,\theta}\right)}{4\left((1-\alpha)\sqrt{\alpha\,\theta\,d}+\alpha\,d\right)^2}=0,
\end{aligned}
\tag{68}
$$

which corresponds to equation (18) on page 10. We will call the value of d that satisfies equation (68) $d^I(\alpha,\theta)$.

D Existence and uniqueness of $d^I(\alpha,\theta)$

Turning to the properties of $H(\alpha,\theta,d)$ we find that

$$
\lim_{d\to 0} H(\alpha,\theta,d)\to\infty, \tag{69}
$$

i.e., $d=0$ could never be a solution for the FOC.
Partial derivation of $H(\alpha,\theta,d)$ subject to d leads (after some tedious algebra) to

$$
\frac{\partial H(\alpha,\theta,d)}{\partial d}=\frac{(1-\alpha)\,\alpha^3\,\Theta}{8\sqrt{\alpha\,\theta\,d}(\alpha\,d+(1-\alpha)\sqrt{\alpha\,\theta\,d})^3}, \tag{70}
$$

with

$$
\begin{aligned}
\Theta=&-\alpha\,d^2+(1-\alpha)\,\theta\left(-4\,\theta(1-\alpha)-(11-\alpha)\sqrt{\alpha\,\theta\,d}\right)\\
&-d\left((1-\alpha)\sqrt{\alpha\,\theta\,d}+3\,\alpha\,\theta(3-\alpha)\right).
\end{aligned}
$$

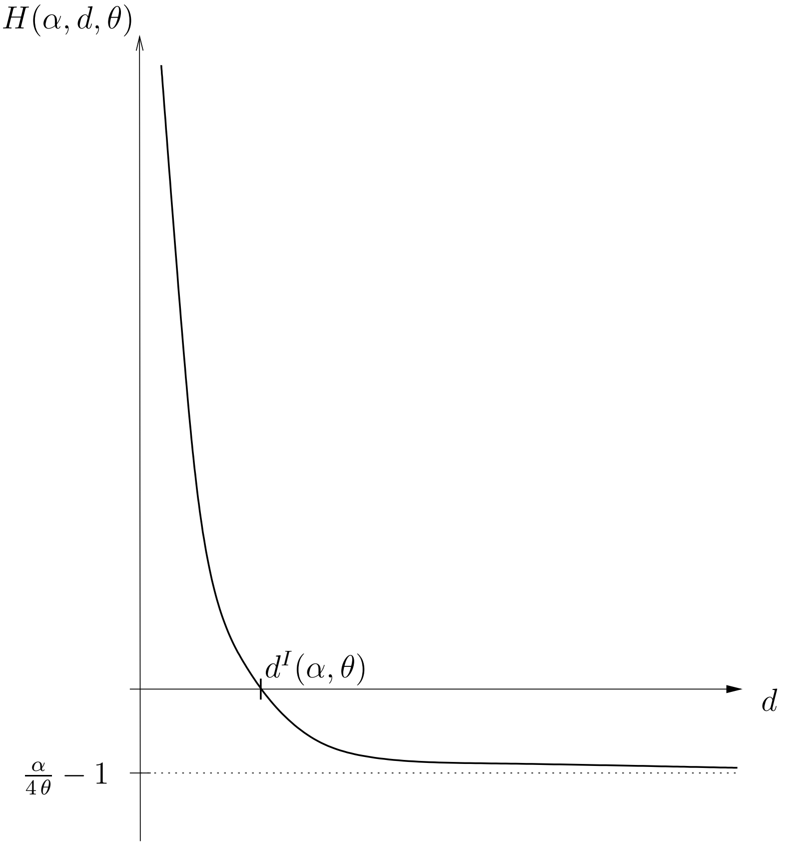

Figure 11: $H(\alpha, d, \theta)$ contingent on d for $\theta > \tilde{\theta}$

Since Θ is unambiguously negative for $d > 0$, $\frac{\partial H(\alpha,\theta,d)}{\partial d}$ is unambiguously negative. Thus, $H(\alpha, \theta, d)$ is a strictly monotonous decreasing and continuous function of d, $\forall\, d > 0$.[23] Moreover, we find that

$$\lim_{d\to\infty} H(\alpha, \theta, d) = -1 + \frac{\alpha}{4\,\theta}. \tag{71}$$

Thus, since $H(\alpha, \theta, d)$ is a strictly monotonous decreasing and continuous function, there exists a unique solution to the FOC, as long as $\theta > \widetilde{\theta}$, with

$$\widetilde{\theta} = \frac{\alpha}{4}. \tag{72}$$

As mentioned before, we will call this solution $d^I(\alpha, \theta)$, with $d^I(\alpha, \theta) \in]0, \infty[$.

E $\bar{\theta} \in [0, 1]$ and $\bar{\theta} > \tilde{\theta}$

Implementing $d = \bar{d}$ in equation 18 and solving it for θ delivers a level of θ as a function of α, which represents the level of the exogenous effectiveness parameter for every $\alpha \in]0, \hat{\alpha}[$, so that the interior and the first corner solution of the second

[23] Since $H(\alpha, \theta, d)$ is continuously differentiable (cf. equation (70)), and since every continuously differentiable function is continuous itself, we know that $H(\alpha, d, \theta)$ is a continous function.

stage apply. That is, if $d^I = \bar{d}$, the restriction on a ($a \geq 0$) is non-binding and the level of d is sufficient to guarantee $a = 0$:

$$\bar{\theta} = \frac{1}{4}\big(2 + \alpha - 2\,\alpha^2\big),$$

which is a continuous and concave function of α, with

$$\bar{\theta}_{\max} = \bar{\theta}\big|_{\alpha=\frac{1}{4}} = \frac{17}{32},$$

so that $\bar{\theta} \in [0,1]$. Moreover, it is easy to verify that

$$\begin{array}{rrcl} & \bar{\theta} & > & \widetilde{\theta} \\ \Leftrightarrow & 1 & > & \alpha. \end{array}$$

Hence, the case separating level of θ always exceeds the minimum level of θ to guarantee the existence of an interior solution in the first stage (existence of $d^I(\alpha,\theta)$) for $\alpha \in]0,\hat{\alpha}[$.

F $d^I > \bar{d}$ for $\theta < \bar{\theta}$ et vice versa

As mentioned in the previous section, $d^I = \bar{d}$ if and only if $\theta = \bar{\theta}$. Since equation (21) on page 12 showed that every investment in defense which exceeds $\bar{d}$ is a waste of resources, we know that the utility maximzing level of d (d^*) is either $d^I \in]0,\bar{d}]$ or $\bar{d}$ with $\bar{d} \leq d^I$ (cf. equations (22) and (23), page 12). In order to proof that

$$d^*(\alpha,\theta) = \begin{cases} \bar{d} & \text{for } \theta \in \left]0,\bar{\theta}\right], \\ d^I < \bar{d} & \text{for } \theta \in \left]\bar{\theta},1\right], \end{cases} \tag{73}$$

we now have to show that

$$\begin{array}{llll} d^I > \bar{d} & \text{for} & \theta < \bar{\theta} & \quad\text{and} \\ d^I < \bar{d} & \text{for} & \theta > \bar{\theta}. & \end{array}$$

We already detected that

1. there is a **unique** level of θ which yields $d^I = \bar{d}$, and that

2. $d^I(\alpha,\theta)$ is a **continuous** function.

Moreover, it is obvious that $\bar{d} = \alpha\,\theta$ is an increasing function of θ. Thus, we only need to show that

$$\left.\frac{\mathrm{d}\, d^I(\alpha,\theta)}{\mathrm{d}\,\theta}\right|_{d^I=\bar{d}} < 0, \tag{74}$$

i.e., at the unique case separating level of θ ($\bar{\theta}$), when $d^I = \bar{d}$, the gradient of the continuous function d^I, with respect to θ, is negative.

Using the implicit function theorem, we know that

$$\frac{\mathrm{d}\, d^I(\alpha,\theta)}{\mathrm{d}\,\theta} = -\frac{\frac{\partial H(\alpha,\theta,d)}{\partial\theta}}{\frac{\partial H(\alpha,\theta,d)}{\partial d}}. \tag{75}$$

After some calculation we get the following expression:

$$\left.\frac{\mathrm{d}\, d^I(\alpha,\theta)}{\mathrm{d}\,\theta}\right|_{\theta=\bar{\theta}, d^I=\bar{d}} = \frac{\alpha^2\left(3(1-\alpha)+2\,\alpha^2\right)}{\alpha\,(2\,\alpha^2+\alpha+1)\;-\;4}, \tag{76}$$

where the enumerator is unambiguously positive and the denominator negative for $\alpha\in]0,\hat{\alpha}[$. Thus, $\left.\frac{\mathrm{d}\, d^I(\alpha,\theta)}{\mathrm{d}\,\theta}\right|_{\theta=\bar{\theta}, d^I=\bar{d}}$ is negative, which concludes the proof.

G Collapse of barter exchange is foreclosed in equilibrium

According to proposition 1, page 9, barter exchange will collapse (2nd corner solution) if the level of defense is equal to or lower than $\underline{d}$ and $\alpha\in\left]\frac{1}{2},\hat{\alpha}\right[$. We will now prove that the collapse of barter exchange is foreclosed in equilibrium (proof of proposition 2.3, page 11).
Under the terms of proposition 1, the interior solution where neither of the restrictions ($a\geq 0$ and $z\in[0,Z]$) are binding, applies as long as

$$\underline{d}\leq d^I\leq\bar{d}. \tag{77}$$

Thus, the following functions apply for $\alpha\in\left]\frac{1}{2},\hat{\alpha}\right[$

$$a(d)=\begin{cases}\frac{-d+\sqrt{d(d+4(1-\alpha)\theta)}}{2\,\theta} & \text{for } d\leq\underline{d},\\ \frac{\sqrt{\alpha\, d\,\theta}-d}{\theta} & \text{for } d\in]\underline{d},\bar{d}[,\\ 0 & \text{for } d\geq\bar{d},\end{cases} \tag{78a}$$

$$z(d)=\begin{cases}Z & \text{for } d\leq\underline{d},\\ \frac{\alpha\sqrt{\theta}Z}{\sqrt{\alpha\, d}+(1-\alpha)\sqrt{\theta}} & \text{for } d\in]\underline{d},\bar{d}\,[,\\ \alpha Z & \text{for } d\geq\bar{d},\end{cases} \tag{78b}$$

with $\bar{d}=\alpha\,\theta$ and $\underline{d}=\frac{(2\,\alpha-1)^2\theta}{\alpha}$. Equation 78b tells us that if d is less than or equal to $\underline{d}$, the amount of nonexported goods (z) equals the initial endowment (Z). Hence, barter exchange collapses. In this case $a(d)$ is reshaped: Now appropriation is the only means to adopt the foreign good.
At first we are interested in the boundary between interior and second corner solution. For this we implement $d=\underline{d}$ in the FOC for an interior solution (equation (18)). Solving this FOC for θ delivers

$$\underline{\theta}=\frac{1+\alpha(\alpha(5-2\,\alpha)-3)}{4(1-2\,\alpha)^2}. \tag{79}$$

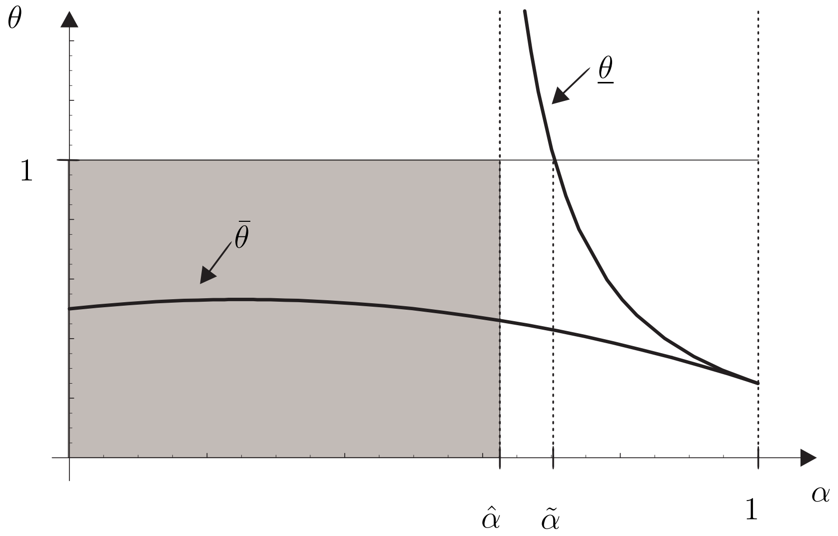

Figure 12: $\bar{\theta}$ and $\underline{\theta}$ contingent on α

Thus, there is a **unique** level of θ for every $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$, so that the interior solution (d^I) equals the level of defense which lets barter exchange vanish ($\underline{d}$). In other words, if $\theta = \underline{\theta}$, the restriction on z ($z \in [0, Z]$) is non-binding and $z = Z$ in equilibrium. Secondly, we will take a closer look at the properties of this case separating level of the exogenous relative effectiveness parameter. Equation 80 and 81 show that for $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$ (the relevant set of the exogenous preference parameter) $\underline{\theta}$ is a monotonously decreasing and convex function of α:

$$\frac{\mathrm{d}\,\underline{\theta}}{\mathrm{d}\,\alpha} = \frac{1 + 2\,\alpha(2(1+\alpha^2) - 3\,\alpha)}{4(1-2\,\alpha)^3}, \tag{80}$$

$$\frac{\mathrm{d}^2\,\underline{\theta}}{\mathrm{d}\,\alpha^2} = \frac{5 + 2\,\alpha}{2(1-2\,\alpha)^4}, \tag{81}$$

where equation (81) is unambiguously positive for the given set of α. Moreover, the enumerator of equation (80) is negative and the denominator is positive $\forall\ \alpha > \frac{1}{2}$.

Equating the two case separating levels of θ delivers

$$\begin{aligned} \underline{\theta} &= \bar{\theta} \\ \alpha &= 1. \end{aligned}$$

Moreover, $\underline{\theta}\big|_{\alpha \to \frac{1}{2}} \to \infty$ and $\bar{\theta}\big|_{\alpha = \frac{1}{2}} = \frac{1}{2}$. Thus,

$$\bar{\theta} < \underline{\theta}, \tag{82}$$

as long as $\alpha < 1$, which is graphically represented in figure 12, with the grey area representing the accredited set of the exogenous relative effectiveness and preference parameter.
To sum things up: Our recent findings are

1. that d^I is a **continuous** function and that $d^I \leq \bar{d}$ as long as $\theta \geq \bar{\theta}$ (cf. section E, page 29),

2. $\bar{\theta} < \underline{\theta}$ for $\alpha < 1$.

3. Moreover, it is easy to verify that $\bar{d} > \underline{d}$ as long as $\alpha \in \left]\frac{1}{3}, 1\right[$.

Thus, if we keep in mind that the necessary condition for the collapse of barter exchange is that $\alpha \in \left]\frac{1}{2}, \hat{\alpha}\right[$, we know that $d^I \in [\underline{d}, \bar{d}]$ as long as $\theta \in \left[\bar{\theta}, \underline{\theta}\right]$.
By definition, the exogenous level of $\theta \in [0, 1]$. Since $\underline{\theta}$ is a monotonically decreasing function of α (cf. equation (80)), equating $\underline{\theta}$ to 1 delivers the minimum level of the exogenous preference parameter, which we will call $\tilde{\alpha}$, to assure that $\underline{\theta} \leq 1$. Since $\tilde{\alpha} \approx 0.7$, this forecloses the emergence of market breakdown for $\alpha \in]\frac{1}{2}, \hat{\alpha}[$ (see figure 12), which concludes the proof.

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